

M 1 a

Trigonometric Functions 1

Time : to : Date Name

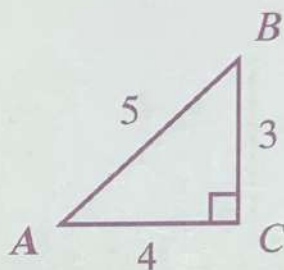
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Given a right triangle, $\triangle ABC$, with C as the right angle,

$$\sin A = \frac{3}{5}$$

$$\cos A = \frac{4}{5}$$

$$\tan A = \frac{3}{4}$$



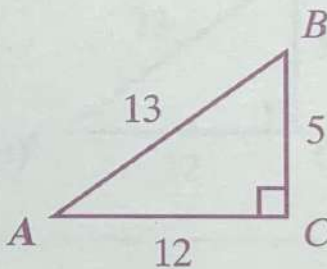
Terminology: $\sin A$, $\cos A$ and $\tan A$ are respectively the *sine*, *cosine* and *tangent* functions of angle A .

Complete and check your answers.

$$\sin A = \frac{5}{13}$$

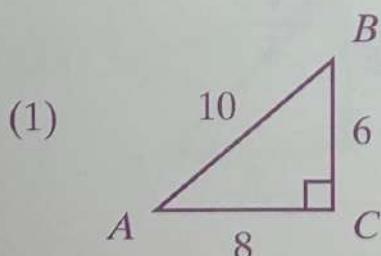
$$\cos A = \frac{12}{13}$$

$$\tan A = \frac{5}{12}$$



Answers: $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}$

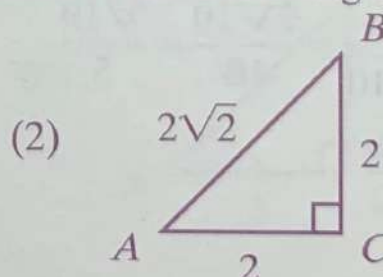
Find the $\sin$, $\cos$ and $\tan$ of angle A in each of the following exercises.



$$\sin A = \frac{6}{10} = \frac{3}{5}$$

$$\cos A = \frac{8}{10} = \frac{4}{5}$$

$$\tan A = \frac{6}{8} = \frac{3}{4}$$

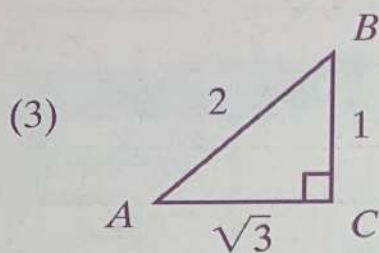


$$\sin A = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos A = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan A = \frac{2}{2} = 1$$

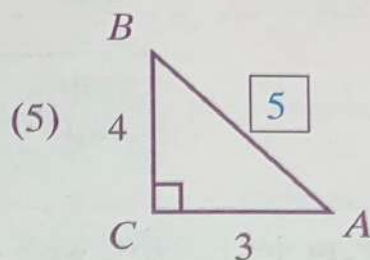
M 1 b



$$\sin A = \frac{1}{2}$$

$$\cos A = \frac{\sqrt{3}}{2}$$

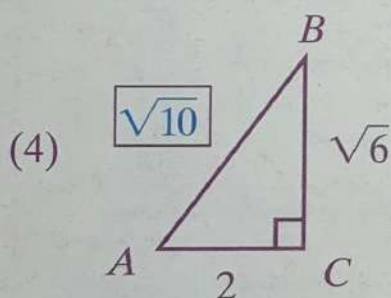
$$\tan A = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$



$$\sin A = \frac{4}{5}$$

$$\cos A = \frac{3}{5}$$

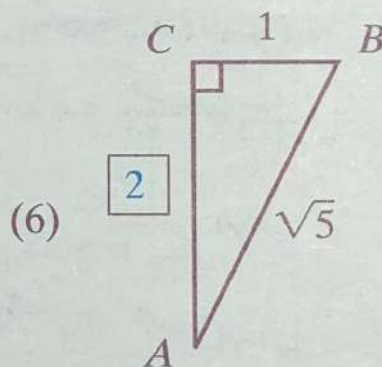
$$\tan A = \frac{4}{3}$$



$$\sin A = \frac{\sqrt{6}}{\sqrt{10}} = \frac{2\sqrt{15}}{10} = \frac{\sqrt{15}}{5}$$

$$\cos A = \frac{2}{\sqrt{10}} = \frac{2\sqrt{10}}{10} = \frac{\sqrt{10}}{5}$$

$$\tan A = \frac{\sqrt{6}}{2}$$



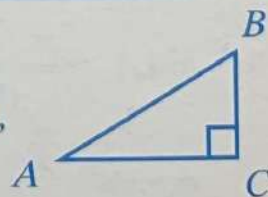
$$\sin A = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos A = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan A = \frac{1}{2}$$

Terminology:

Given a right triangle, $\triangle ABC$ with C as the right angle, AC is the side *adjacent* to angle A , BC is the side *opposite* of angle A , AB is the *hypotenuse*.



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan A = \frac{\text{opposite}}{\text{adjacent}}$$

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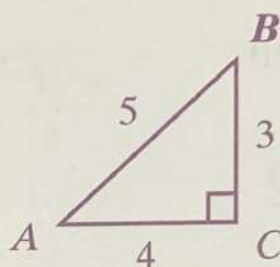
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Given a right triangle, $\triangle ABC$, with C as the right angle,

$$\sin B = \frac{4}{5}$$

$$\cos B = \frac{3}{5}$$

$$\tan B = \frac{4}{3}$$

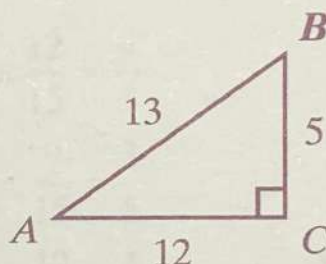


Complete and check your answers.

$$\sin B = \frac{12}{13}$$

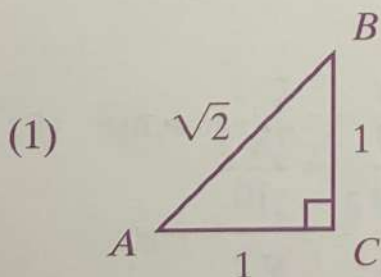
$$\cos B = \frac{5}{13}$$

$$\tan B = \frac{12}{5}$$



Answers: $\frac{12}{13}, \frac{5}{13}, \frac{12}{5}$

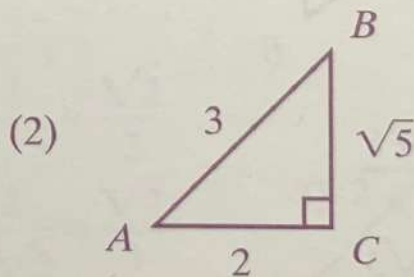
Find the sin, cos and tan of angle B in each of the following exercises.



$$\sin B = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos B = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan B = \frac{1}{1} = 1$$

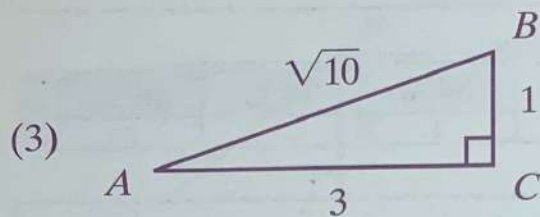


$$\sin B = \frac{2}{3}$$

$$\cos B = \frac{\sqrt{5}}{3}$$

$$\tan B = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

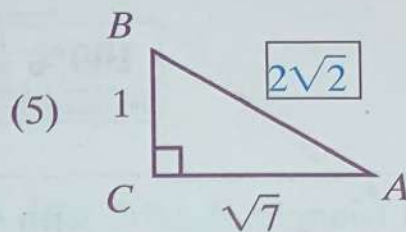
M 2 b



$$\sin B = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos B = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

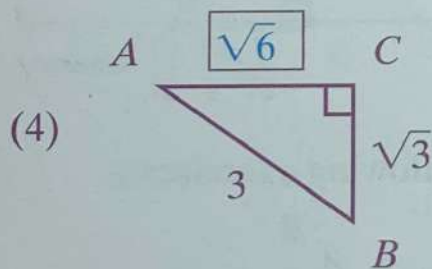
$$\tan B = \frac{3}{1} = 3$$



$$\sin B = \frac{\sqrt{7}}{2\sqrt{2}} = \frac{\sqrt{14}}{4}$$

$$\cos B = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

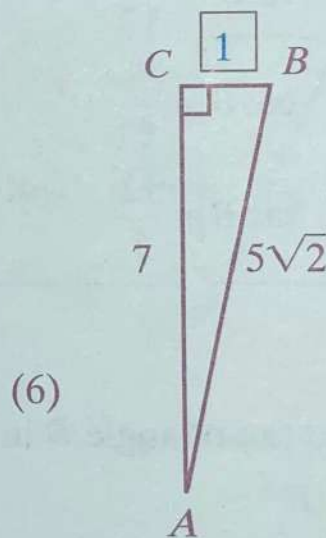
$$\tan B = \frac{\sqrt{7}}{1} = \sqrt{7}$$



$$\sin B = \frac{\sqrt{6}}{3}$$

$$\cos B = \frac{\sqrt{3}}{3}$$

$$\tan B = \frac{\sqrt{6}}{\sqrt{3}} = \frac{\sqrt{18}}{3} = \frac{3\sqrt{2}}{3} = \sqrt{2}$$



$$\sin B = \frac{7}{5\sqrt{2}} = \frac{7\sqrt{2}}{10}$$

$$\cos B = \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$$

$$\tan B = \frac{7}{1} = 7$$

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Find the sin, cos and tan of angles **A** and **B** as shown in the example.

Ex.

$$\sin A = \frac{12}{13}$$

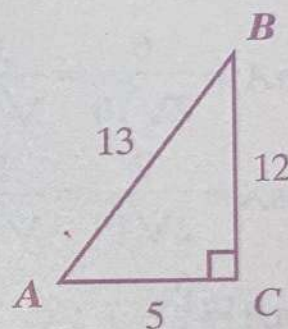
$$\cos A = \frac{5}{13}$$

$$\tan A = \frac{12}{5}$$

$$\sin B = \frac{5}{13}$$

$$\cos B = \frac{12}{13}$$

$$\tan B = \frac{5}{12}$$



$$(1) \sin A = \frac{9}{15} = \frac{3}{5}$$

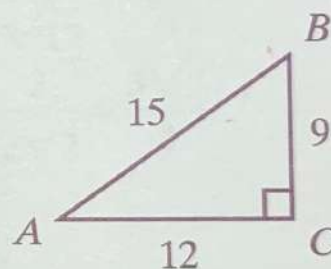
$$\cos A = \frac{12}{15} = \frac{4}{5}$$

$$\tan A = \frac{9}{12} = \frac{3}{4}$$

$$\sin B = \frac{12}{15} = \frac{4}{5}$$

$$\cos B = \frac{9}{15} = \frac{3}{5}$$

$$\tan B = \frac{12}{9} = \frac{4}{3}$$



$$(2) \sin A = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

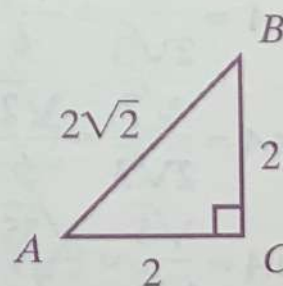
$$\cos A = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan A = \frac{2}{2} = 1$$

$$\sin B = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

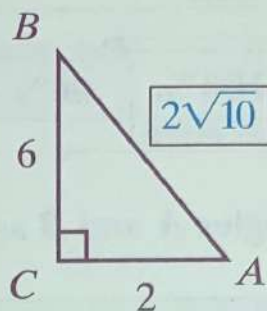
$$\cos B = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan B = \frac{2}{2} = 1$$



M 3 b

(3)



$$\sin A = \frac{6}{2\sqrt{10}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos A = \frac{2}{2\sqrt{10}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

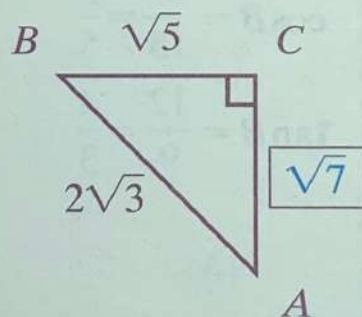
$$\tan A = \frac{6}{2} = 3$$

$$\sin B = \frac{2}{2\sqrt{10}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos B = \frac{6}{2\sqrt{10}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\tan B = \frac{2}{6} = \frac{1}{3}$$

(4)



$$\sin A = \frac{\sqrt{5}}{2\sqrt{3}} = \frac{\sqrt{15}}{6}$$

$$\cos A = \frac{\sqrt{7}}{2\sqrt{3}} = \frac{\sqrt{21}}{6}$$

$$\tan A = \frac{\sqrt{5}}{\sqrt{7}} = \frac{\sqrt{35}}{7}$$

$$\sin B = \frac{\sqrt{7}}{2\sqrt{3}} = \frac{\sqrt{21}}{6}$$

$$\cos B = \frac{\sqrt{5}}{2\sqrt{3}} = \frac{\sqrt{15}}{6}$$

$$\tan B = \frac{\sqrt{7}}{\sqrt{5}} = \frac{\sqrt{35}}{5}$$

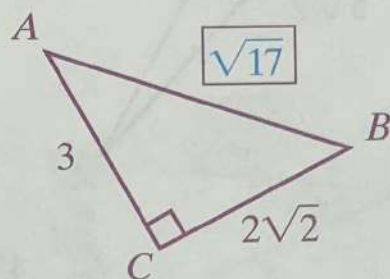
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Find the sin, cos and tan of angles A and B .

(1)



$$\sin A = \frac{2\sqrt{2}}{\sqrt{17}} = \frac{2\sqrt{34}}{17}$$

$$\cos A = \frac{3}{\sqrt{17}} = \frac{3\sqrt{17}}{17}$$

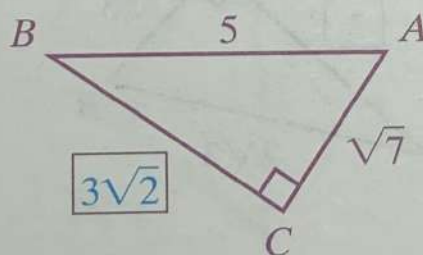
$$\tan A = \frac{2\sqrt{2}}{3}$$

$$\sin B = \frac{3}{\sqrt{17}} = \frac{3\sqrt{17}}{17}$$

$$\cos B = \frac{2\sqrt{2}}{\sqrt{17}} = \frac{2\sqrt{34}}{17}$$

$$\tan B = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

(2)



$$\sin A = \frac{3\sqrt{2}}{5}$$

$$\cos A = \frac{\sqrt{7}}{5}$$

$$\tan A = \frac{3\sqrt{2}}{\sqrt{7}} = \frac{3\sqrt{14}}{7}$$

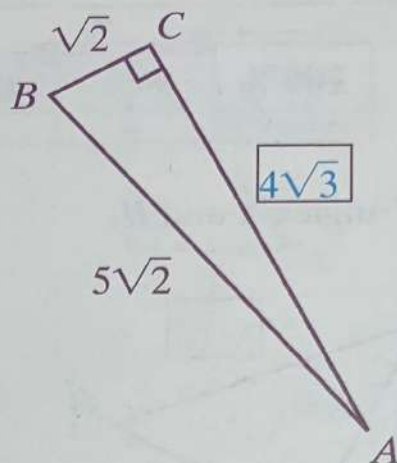
$$\sin B = \frac{\sqrt{7}}{5}$$

$$\cos B = \frac{3\sqrt{2}}{5}$$

$$\tan B = \frac{\sqrt{7}}{3\sqrt{2}} = \frac{\sqrt{14}}{6}$$

M 4 b

(3)



$$\sin A = \frac{\sqrt{2}}{5\sqrt{2}} = \frac{1}{5}$$

$$\cos A = \frac{4\sqrt{3}}{5\sqrt{2}} = \frac{4\sqrt{6}}{10} = \frac{2\sqrt{6}}{5}$$

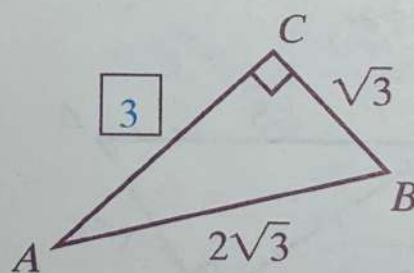
$$\tan A = \frac{\sqrt{2}}{4\sqrt{3}} = \frac{\sqrt{6}}{12}$$

$$\sin B = \frac{4\sqrt{3}}{5\sqrt{2}} = \frac{4\sqrt{6}}{10} = \frac{2\sqrt{6}}{5}$$

$$\cos B = \frac{\sqrt{2}}{5\sqrt{2}} = \frac{1}{5}$$

$$\tan B = \frac{4\sqrt{3}}{\sqrt{2}} = \frac{4\sqrt{6}}{2} = 2\sqrt{6}$$

(4)



$$\sin A = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$$

$$\cos A = \frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$$\tan A = \frac{\sqrt{3}}{3}$$

$$\sin B = \frac{3}{2\sqrt{3}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$$\cos B = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$$

$$\tan B = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

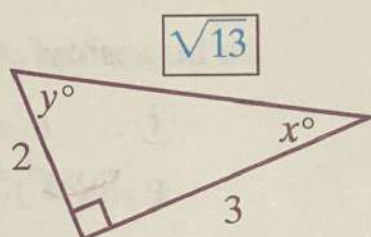
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Find the sin, cos and tan of the indicated angles in each of the following exercises.

(1)



$$\sin x^\circ = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\sin y^\circ = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

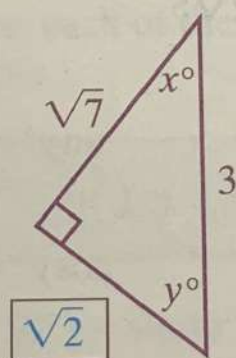
$$\cos x^\circ = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\cos y^\circ = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\tan x^\circ = \frac{2}{3}$$

$$\tan y^\circ = \frac{3}{2}$$

(2)



$$\sin x^\circ = \frac{\sqrt{2}}{3}$$

$$\sin y^\circ = \frac{\sqrt{7}}{3}$$

$$\cos x^\circ = \frac{\sqrt{7}}{3}$$

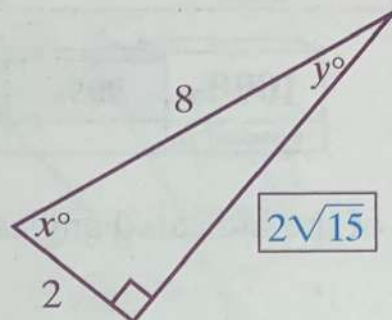
$$\cos y^\circ = \frac{\sqrt{2}}{3}$$

$$\tan x^\circ = \frac{\sqrt{2}}{\sqrt{7}} = \frac{\sqrt{14}}{7}$$

$$\tan y^\circ = \frac{\sqrt{7}}{\sqrt{2}} = \frac{\sqrt{14}}{2}$$

M 5 b

(3)



$$\sin x^\circ = \frac{2\sqrt{15}}{8} = \frac{\sqrt{15}}{4}$$

$$\sin y^\circ = \frac{2}{8} = \frac{1}{4}$$

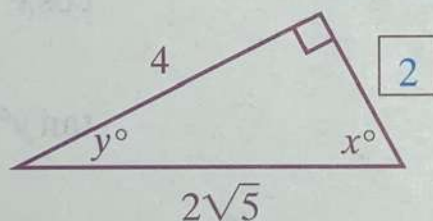
$$\cos x^\circ = \frac{2}{8} = \frac{1}{4}$$

$$\cos y^\circ = \frac{2\sqrt{15}}{8} = \frac{\sqrt{15}}{4}$$

$$\tan x^\circ = \frac{2\sqrt{15}}{2} = \sqrt{15}$$

$$\tan y^\circ = \frac{2}{2\sqrt{15}} = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$$

(4)



$$\sin x^\circ = \frac{4}{2\sqrt{5}} = \frac{4\sqrt{5}}{10} = \frac{2\sqrt{5}}{5}$$

$$\sin y^\circ = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos x^\circ = \frac{2}{2\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos y^\circ = \frac{4}{2\sqrt{5}} = \frac{4\sqrt{5}}{10} = \frac{2\sqrt{5}}{5}$$

$$\tan x^\circ = \frac{4}{2} = 2$$

$$\tan y^\circ = \frac{2}{4} = \frac{1}{2}$$

Note Summary:

From exercises (1)–(4), note that $\sin x^\circ = \cos y^\circ$

$$\cos x^\circ = \sin y^\circ$$

$$\tan x^\circ = \frac{1}{\tan y^\circ}$$

Answers: $y^\circ, \tan y^\circ$

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From the Triangle-Sum Theorem in Level I, $m\angle A + m\angle B + m\angle C = 180^\circ$.
Therefore, given a right triangle, $\triangle ABC$, with C as the right angle,

$$m\angle A + m\angle B = 90^\circ$$

Given angle A , B can be expressed as:

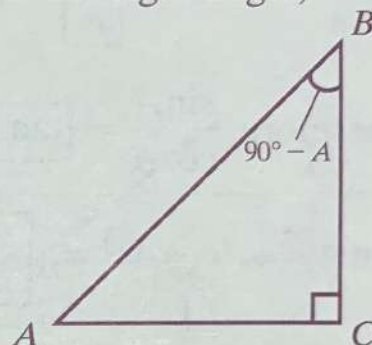
$$B = 90^\circ - A \dots \textcircled{1}$$

Since $\sin A = \cos B$

Substituting $\textcircled{1}$, $\sin A = \cos(90^\circ - A)$

Similarly, since $\cos A = \sin B$ and $\tan A = \frac{1}{\tan B}$,

Substituting $\textcircled{1}$, $\cos A = \sin(90^\circ - A)$ and $\tan A = \frac{1}{\tan(90^\circ - A)}$



Answers: $\sin(90^\circ - A)$, $\tan(90^\circ - A)$

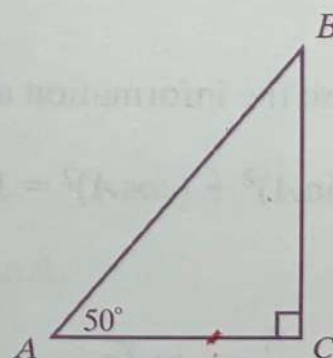
1. Given $A = 50^\circ$, express each of the following functions in terms of 40° , as shown in the example.

Ex.

$$\sin 50^\circ = \cos(90^\circ - 50^\circ) = \cos 40^\circ$$

(1) $\cos 50^\circ = \sin(90^\circ - 50^\circ) = \sin 40^\circ$

(2) $\tan 50^\circ = \frac{1}{\tan(90^\circ - 50^\circ)} = \frac{1}{\tan 40^\circ}$



2. Given $A = 20^\circ$, express each of the following functions in terms of 70° .

(1) $\sin 20^\circ = \cos(90^\circ - 20^\circ) = \cos 70^\circ$

(2) $\cos 20^\circ = \sin(90^\circ - 20^\circ) = \sin 70^\circ$

(3) $\tan 20^\circ = \frac{1}{\tan(90^\circ - 20^\circ)} = \frac{1}{\tan 70^\circ}$

$$\frac{\sin A}{\cos A} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{b} \quad \text{and} \quad \tan A = \frac{a}{b}$$

Therefore, $\frac{\sin A}{\cos A} = \boxed{\tan A} \dots \textcircled{1}$

$$\begin{aligned}(\sin A)^2 + (\cos A)^2 &= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 \\&= \frac{a^2 + b^2}{c^2} \\&= \frac{c^2}{c^2} = 1\end{aligned}$$

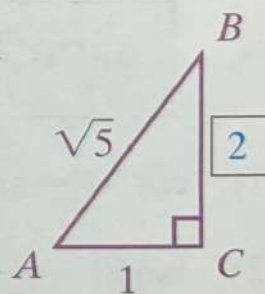
From the Pythagorean Theorem,
 $a^2 + b^2 = c^2$

Therefore, $(\sin A)^2 + (\cos A)^2 = \boxed{1} \dots \textcircled{2}$

Answers: $\frac{c}{a}, \frac{b}{a}, \tan A, \frac{c}{a}, \frac{c}{b}, a^2 + b^2, c^2, 1, 1$

3. Use the information above to show that $\frac{\sin A}{\cos A} = \tan A$, and

$(\sin A)^2 + (\cos A)^2 = 1$ for the given triangle on the right.



$$(1) \quad \frac{\sin A}{\cos A} = \frac{\frac{2\sqrt{5}}{5}}{\frac{\sqrt{5}}{5}} = 2 \quad \text{and} \quad \tan A = 2 \quad \therefore \frac{\sin A}{\cos A} = \tan A$$

$$(2) \quad (\sin A)^2 + (\cos A)^2 = \left(\frac{2\sqrt{5}}{5}\right)^2 + \left(\frac{\sqrt{5}}{5}\right)^2 = \frac{4}{5} + \frac{1}{5} = 1$$

$$\therefore (\sin A)^2 + (\cos A)^2 = 1$$

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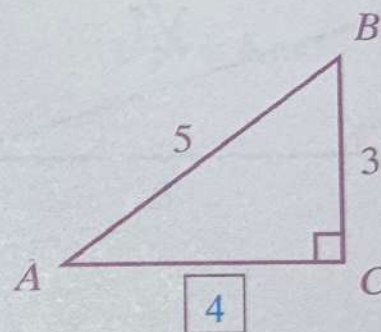
Ex.

Given $\sin A = \frac{3}{5}$, find the values of $\cos A$ and $\tan A$.

[Sol] $AC = \sqrt{5^2 - 3^2} = \boxed{4}$

$$\cos A = \frac{\boxed{4}}{5}$$

$$\tan A = \frac{3}{\boxed{4}}$$



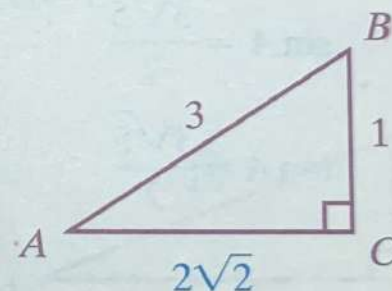
Answers: all 4

(1) Given $\sin A = \frac{1}{3}$, find the values of $\cos A$ and $\tan A$.

[Sol] $AC = \sqrt{3^2 - 1^2} = 2\sqrt{2}$

$$\cos A = \frac{2\sqrt{2}}{3}$$

$$\tan A = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

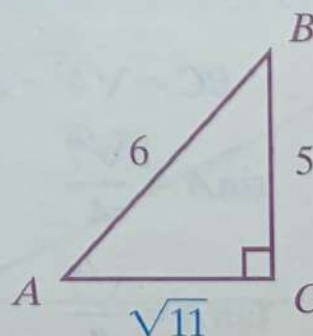


(2) Given $\sin A = \frac{5}{6}$, find the values of $\cos A$ and $\tan A$.

[Sol] $AC = \sqrt{6^2 - 5^2} = \sqrt{11}$

$$\cos A = \frac{\sqrt{11}}{6}$$

$$\tan A = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$



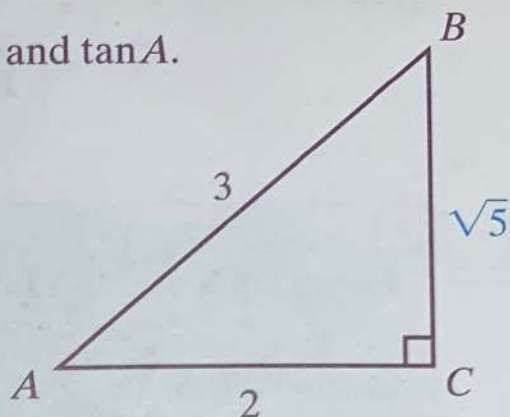
M 7 b

- (3) Given $\cos A = \frac{2}{3}$, find the values of $\sin A$ and $\tan A$.

[Sol] $BC = \sqrt{3^2 - 2^2} = \sqrt{5}$

$$\sin A = \frac{\sqrt{5}}{3}$$

$$\tan A = \frac{\sqrt{5}}{2}$$

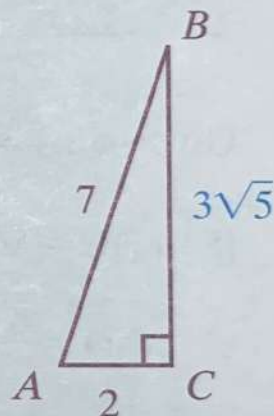


- (4) Given $\cos A = \frac{2}{7}$, find the values of $\sin A$ and $\tan A$.

[Sol] $BC = \sqrt{7^2 - 2^2} = 3\sqrt{5}$

$$\sin A = \frac{3\sqrt{5}}{7}$$

$$\tan A = \frac{3\sqrt{5}}{2}$$



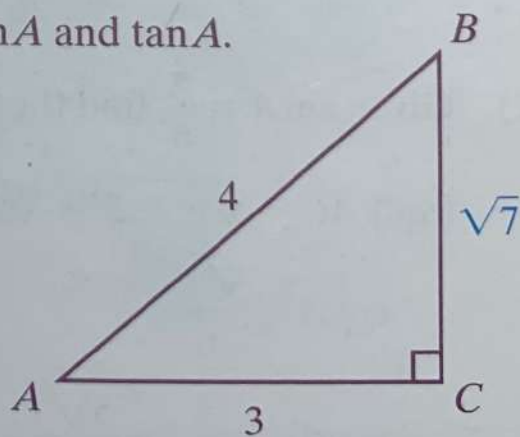
- (5) Given $\cos A = 0.75$, find the values of $\sin A$ and $\tan A$.

[Sol] $\cos A = 0.75 = \frac{3}{4}$

$$BC = \sqrt{4^2 - 3^2} = \sqrt{7}$$

$$\sin A = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\sqrt{7}}{3}$$



Trigonometric Functions 1

Time : to : Date Name

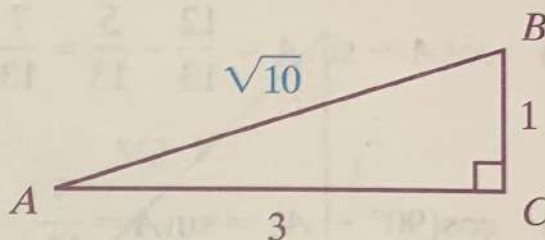
100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

1. Given $\tan A = \frac{1}{3}$, find the values of $\sin A$ and $\cos A$.

[Sol] $AB = \sqrt{3^2 + 1^2} = \sqrt{10}$

$$\sin A = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos A = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$



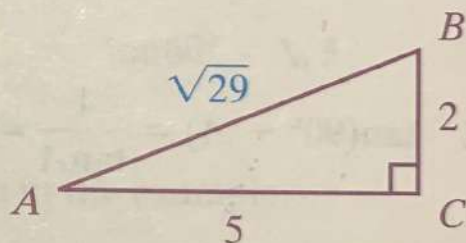
2. Given $\tan A = 0.4$, find the values of $\sin A$ and $\cos A$.

[Sol] $\tan A = 0.4 = \frac{2}{5}$

$$AB = \sqrt{5^2 + 2^2} = \sqrt{29}$$

$$\sin A = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$$

$$\cos A = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$$

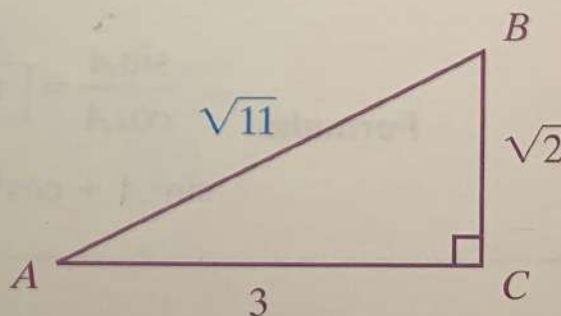


3. Given $\tan A = \frac{\sqrt{2}}{3}$, find the values of $\sin A$ and $\cos A$.

[Sol] $AB = \sqrt{(\sqrt{2})^2 + 3^2} = \sqrt{11}$

$$\sin A = \frac{\sqrt{2}}{\sqrt{11}} = \frac{\sqrt{22}}{11}$$

$$\cos A = \frac{3}{\sqrt{11}} = \frac{3\sqrt{11}}{11}$$



M 8 b

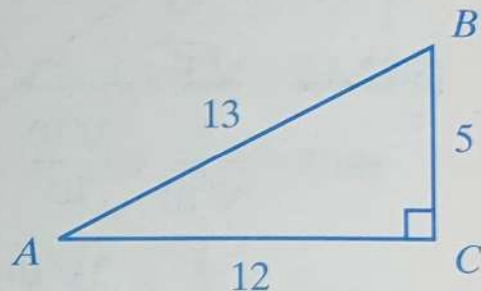
4. Given $\sin A = \frac{5}{13}$, evaluate the following expressions.

$$(1) \quad \cos A = \sin(90^\circ - A) = \frac{12}{13}$$

$$[AC = \sqrt{(13)^2 - 5^2} = 12]$$

$$(2) \quad \cos A - \sin A = \frac{12}{13} - \frac{5}{13} = \frac{7}{13}$$

$$(3) \quad \cos(90^\circ - A) = \sin A = \frac{5}{13}$$



$$(4) \quad \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{13} \cdot \frac{13}{12} = \frac{5}{12}$$

$$(5) \quad \tan A = \frac{5}{12}$$

$$(6) \quad \tan(90^\circ - A) = \frac{1}{\tan A} = \frac{12}{5}$$

$$(7) \quad \sin^2 A + \cos^2 A = \left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = \frac{25}{169} + \frac{144}{169} = \frac{169}{169} = 1$$

Complete and check the Formulas.

Formulas $\frac{\sin A}{\cos A} = \boxed{\tan} A$ From exercises (4) and (5)

$\sin^2 A + \cos^2 A = \boxed{1}$ From exercise (7)

Answers: tan, 1

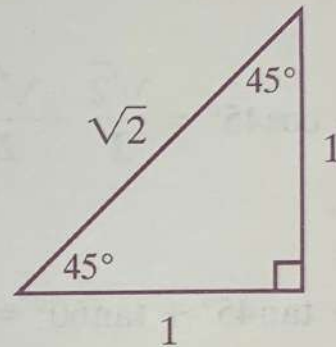
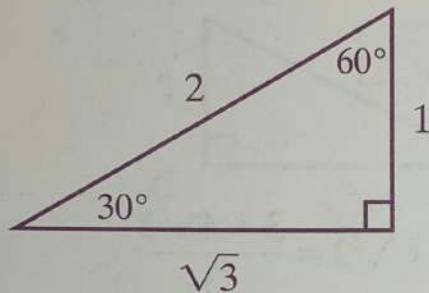
Notation: $\sin^2 A = (\sin A)^2$

Trigonometric Functions 1

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2~3	4~5	6~

1. Using the figures below, find the value of each of the trigonometric functions of 30° , 45° and 60° .



$$\begin{array}{lll}
 (1) \quad \sin 30^\circ = \frac{1}{2} & (2) \quad \sin 45^\circ = \frac{\sqrt{2}}{2} & (3) \quad \sin 60^\circ = \frac{\sqrt{3}}{2} \\
 \cos 30^\circ = \frac{\sqrt{3}}{2} & \cos 45^\circ = \frac{\sqrt{2}}{2} & \cos 60^\circ = \frac{1}{2} \\
 \tan 30^\circ = \frac{\sqrt{3}}{3} & \tan 45^\circ = 1 & \tan 60^\circ = \sqrt{3}
 \end{array}$$

2. Evaluate the following expressions as shown in the example.

Ex.

$$\sin 30^\circ \cos 60^\circ = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$(1) \quad \sin 45^\circ \tan 45^\circ = \frac{\sqrt{2}}{2} \cdot 1 = \frac{\sqrt{2}}{2}$$

$$(2) \quad \sin 30^\circ + \sin 60^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

$$(3) \quad \frac{\cos 45^\circ}{\cos 30^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$(4) \quad \tan 60^\circ \tan 30^\circ = \sqrt{3} \cdot \frac{\sqrt{3}}{3} = 1$$

M 9 b

$$(5) \quad \sin 30^\circ + \sin 45^\circ + \sin 60^\circ = \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{2} + \sqrt{3}}{2}$$

$$(6) \quad \cos 30^\circ + \cos 45^\circ + \cos 60^\circ = \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{1 + \sqrt{2} + \sqrt{3}}{2}$$

$$(7) \quad \sin 45^\circ - \cos 45^\circ = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$$

$$(8) \quad \tan 30^\circ + \tan 45^\circ + \tan 60^\circ = \frac{\sqrt{3}}{3} + 1 + \sqrt{3} = \frac{3 + 4\sqrt{3}}{3}$$

$$(9) \quad \sin 30^\circ \tan 45^\circ + \cos 45^\circ \tan 45^\circ = \frac{1}{2} \cdot 1 + \frac{\sqrt{2}}{2} \cdot 1 = \frac{1 + \sqrt{2}}{2}$$

$$(10) \quad \frac{1}{\sin 30^\circ} - \frac{1}{\cos 60^\circ} \cdot \frac{1}{\tan 45^\circ} = 2 - 2 = 0$$

$$(11) \quad \frac{1}{\tan 30^\circ} + \tan 45^\circ = \sqrt{3} + 1$$

$$(12) \quad \frac{\sin 60^\circ}{\cos 30^\circ} + \frac{\sin 45^\circ}{\cos 45^\circ} = 1 + 1 = 2$$

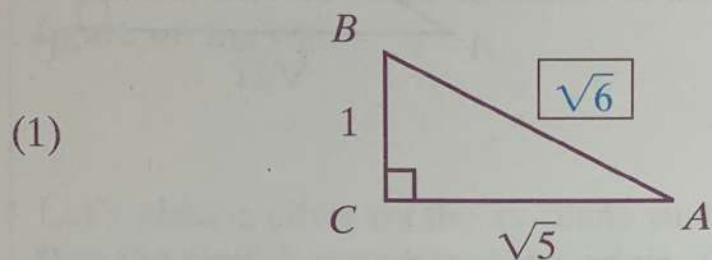
$$(13) \quad \frac{1}{1 + \sin 45^\circ} = \frac{1}{1 + \frac{\sqrt{2}}{2}} = \frac{1}{\frac{2 + \sqrt{2}}{2}} = \frac{2}{2 + \sqrt{2}} = \frac{4 - 2\sqrt{2}}{2} = 2 - \sqrt{2}$$

$$(14) \quad \frac{1 + \tan 30^\circ}{1 - \tan 30^\circ} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{\frac{3 + \sqrt{3}}{3}}{\frac{3 - \sqrt{3}}{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}} = \frac{12 + 6\sqrt{3}}{6} = 2 + \sqrt{3}$$

Time : to : Date Name

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1. Find the sin, cos and tan of the indicated angles in each of the following exercises.



$$\sin A = \frac{\sqrt{6}}{6}$$

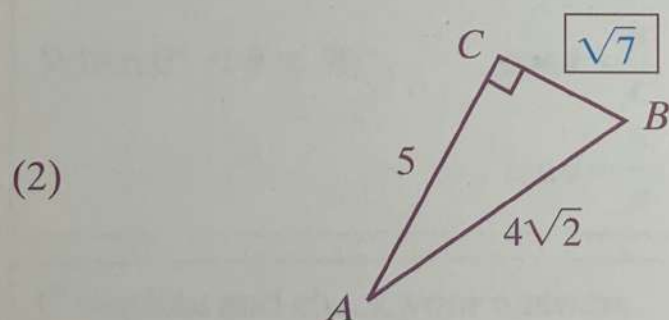
$$\cos A = \frac{\sqrt{30}}{6}$$

$$\tan A = \frac{\sqrt{5}}{5}$$

$$\sin B = \frac{\sqrt{30}}{6}$$

$$\cos B = \frac{\sqrt{6}}{6}$$

$$\tan B = \sqrt{5}$$



$$\sin A = \frac{\sqrt{7}}{4\sqrt{2}} = \frac{\sqrt{14}}{8}$$

$$\cos A = \frac{5}{4\sqrt{2}} = \frac{5\sqrt{2}}{8}$$

$$\tan A = \frac{\sqrt{7}}{5}$$

$$\sin B = \frac{5}{4\sqrt{2}} = \frac{5\sqrt{2}}{8}$$

$$\cos B = \frac{\sqrt{7}}{4\sqrt{2}} = \frac{\sqrt{14}}{8}$$

$$\tan B = \frac{5}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$$

M 10 b

2. Given $\sin A = \frac{2}{5}$, evaluate the following expressions.

$$(1) \cos A = \frac{\sqrt{21}}{5}$$

$$(2) \tan A = \frac{2}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$

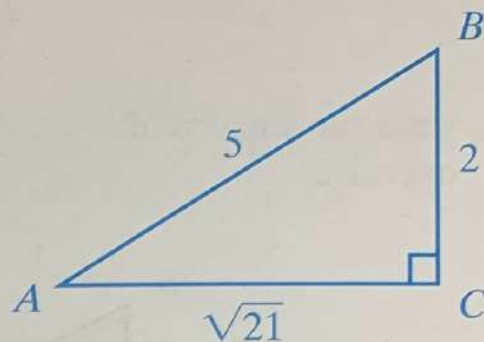
$$(3) \sin(90^\circ - A) = \cos A = \frac{\sqrt{21}}{5}$$

$$(4) \cos(90^\circ - A) = \sin A = \frac{2}{5}$$

$$(5) \tan(90^\circ - A) = \frac{1}{\tan A} = \frac{\sqrt{21}}{2}$$

$$(6) \frac{\sin A}{\cos A} = \tan A = \frac{2\sqrt{21}}{21}$$

$$(7) \sin^2 A + \cos^2 A = 1$$



3. Evaluate the following expressions.

$$(1) \sin 60^\circ \tan 45^\circ = \frac{\sqrt{3}}{2} \cdot 1 = \frac{\sqrt{3}}{2}$$

$$(2) \frac{\cos 30^\circ}{\sin 30^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$(3) \sin 45^\circ + \cos 45^\circ = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$$

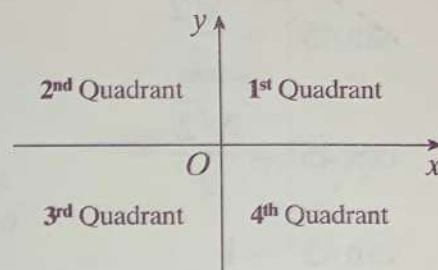
$$(4) \frac{\tan 30^\circ}{\tan 60^\circ} + \cos 60^\circ \sin 30^\circ = \frac{\frac{\sqrt{3}}{3}}{\sqrt{3}} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

Trigonometric Functions 2

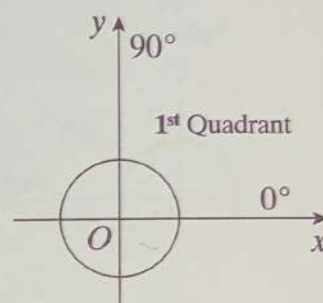
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(mistakes) 0	-	-	-	1~

The axes of the xy -plane form four regions, called *Quadrants*, numbered in a counterclockwise order as shown in the figure on the right.



Let's place a circle on the xy -plane such that the circle's center is at the origin, O . Since a circle measures 360° , we can label the **1st Quadrant** to be from 0° to 90° .



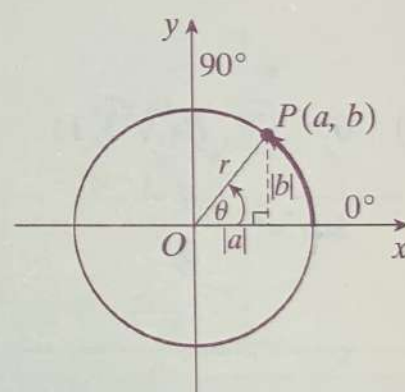
Letting r be the radius of the circle and $P(a, b)$ be a point on the circle, and letting θ be the shortest angle formed from the x -axis to OP ,

$$\sin \theta = \frac{b}{r}$$

When $0^\circ < \theta < 90^\circ$,

$$\cos \theta = \frac{a}{r}$$

$$\tan \theta = \frac{b}{a}$$



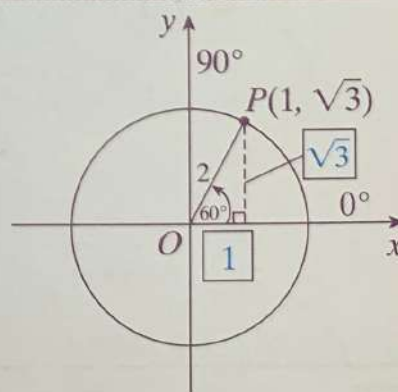
Complete and check your answers.

Given $\theta = 60^\circ$,

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$



Answers: functions: $\frac{\sqrt{3}}{2}, \frac{1}{2}, \sqrt{3}$; graph: 1, $\sqrt{3}$

Note: In this set, $|a|$ and $|b|$ are used to label the lengths of the legs of a right triangle.

M 11 b

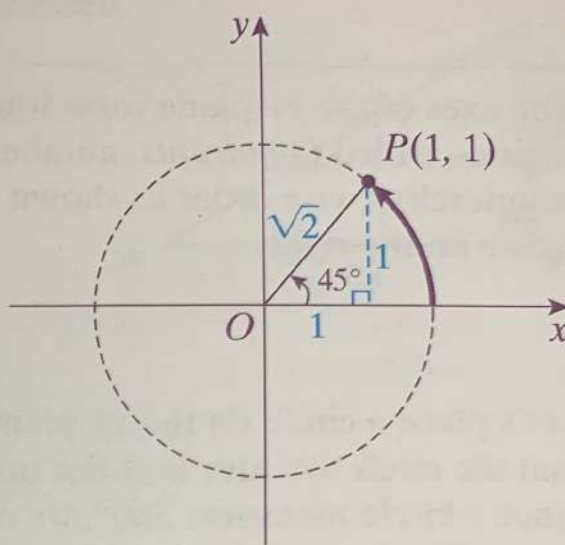
In each exercise, draw the diagram, and use it to evaluate the trigonometric functions of the given angle.

(1) $\theta = 45^\circ$, $P(1, 1)$

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = 1$$

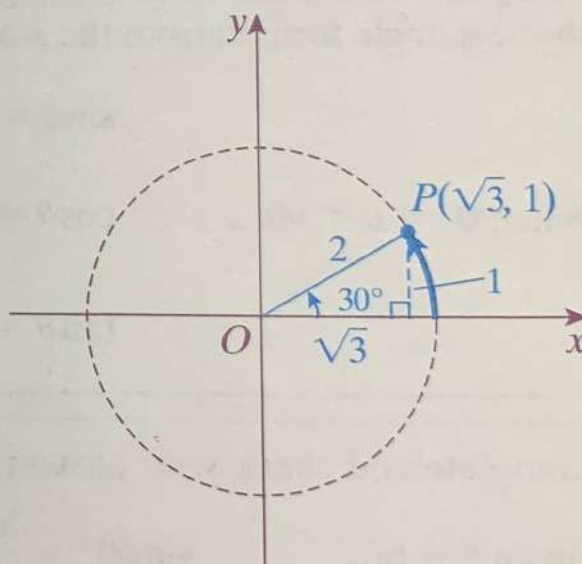


(2) $\theta = 30^\circ$, $P(\sqrt{3}, 1)$

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{\sqrt{3}}{3}$$



Note:

In the 1st Quadrant, both of the coordinates of point $P(a, b)$ have a positive value, i.e. $a > 0$ and $b > 0$. The radius of the circle, r , which in these exercises forms the hypotenuse of the right triangle, is always positive, i.e. $r > 0$.

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	-	-	1~

If the 1st Quadrant measures from 0° to 90° , we can label the **2nd Quadrant** to be from 90° to 180° .

Given an angle θ , where $90^\circ < \theta < 180^\circ$, we can draw an arc, to mark the measure from 0° to θ° ending at point $P(a, b)$.

Letting α be the shortest angle formed from the x -axis to OP ,

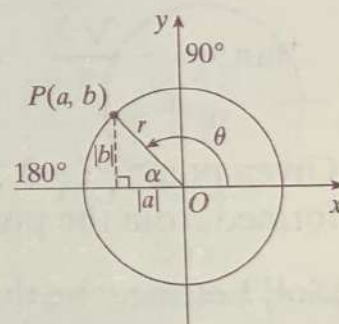
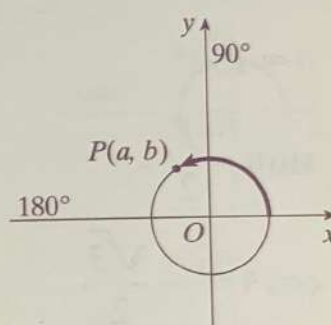
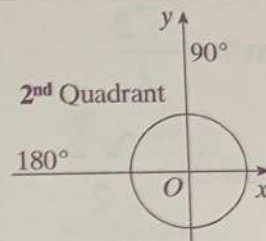
To determine the trigonometric values of θ , we draw a right triangle, using angle α .

$$\sin \theta = \sin \alpha = \frac{b}{r}$$

$$\text{When } 90^\circ < \theta < 180^\circ, \quad \cos \theta = -\cos \alpha = -\frac{a}{r}$$

$$\tan \theta = -\tan \alpha = -\frac{b}{a}$$

*Note that in the 2nd Quadrant, $a < 0$ and $b > 0$.



Ex.

Given $\theta = 120^\circ$, determine the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

[Sol] Sketching the location of θ ,

From the graph, $\alpha = 60^\circ$.

Since we are working in the 2nd Quadrant,

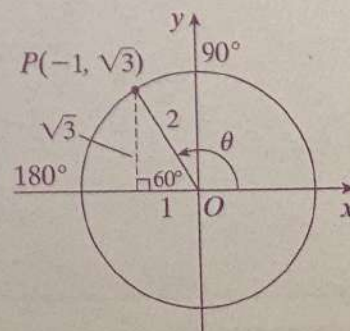
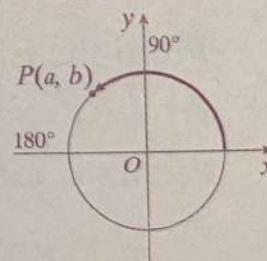
$$a = -1 \text{ and } b = \sqrt{3}$$

Therefore,

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$



M 12 b

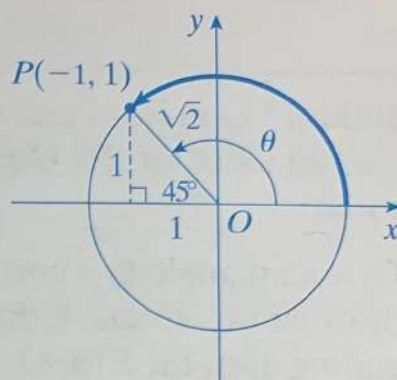
1. In each exercise, draw the diagram, and use it to evaluate the trigonometric functions of the given angle.

(1) $\theta = 135^\circ$

$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$

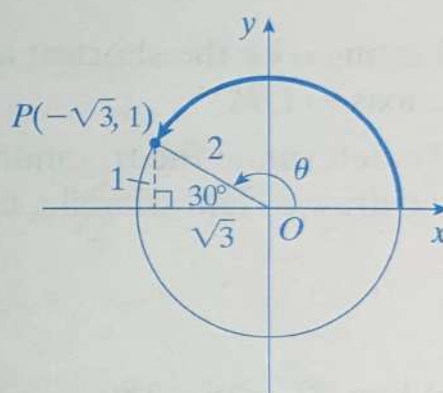


(2) $\theta = 150^\circ$

$$\sin \theta = \frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$



2. Given point $P(-2, 2\sqrt{3})$, find the value of the sin, cos and tan of angle θ , formed from the positive x -axis (moving counterclockwise) to OP .

[Sol] Letting θ be the angle formed from the positive x -axis (moving counterclockwise) to OP , and letting α be the shortest angle formed from the x -axis to OP ,

From the triangle,

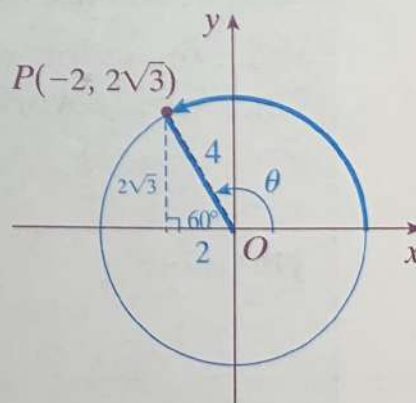
$$OP = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4$$

From the ratio of the sides, $\alpha = 60^\circ$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$



M 13 a

Trigonometric Functions 2

Time : to : Date Name

100%	90%	80%	70%	60%~
(mistakes) 0	-	-	-	1~

Let's label the **3rd Quadrant** to be from 180° to 270° .

Given an angle θ , where $180^\circ < \theta < 270^\circ$, we can draw an arc, to mark the measure from 0° to θ° ending at point $P(a, b)$.

Letting α be the shortest angle formed from the x -axis to OP ,

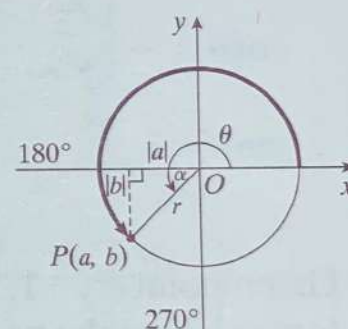
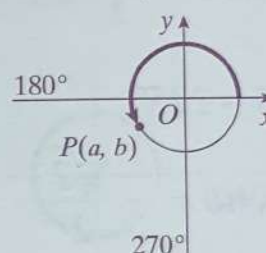
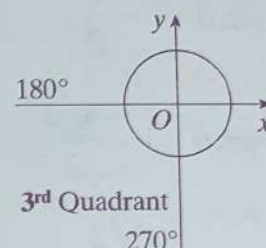
To determine the trigonometric values of θ , we draw a right triangle, using angle α .

$$\sin \theta = -\sin \alpha = \frac{b}{r}$$

$$\text{When } 180^\circ < \theta < 270^\circ, \quad \cos \theta = -\cos \alpha = \frac{a}{r}$$

$$\tan \theta = \tan \alpha = \frac{b}{a}$$

*Note that in the 3rd Quadrant, $a < 0$ and $b < 0$.



Ex.

Given $\theta = 240^\circ$ determine the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

[Sol] Sketching the location of θ ,

From the graph, $\alpha = 60^\circ$.

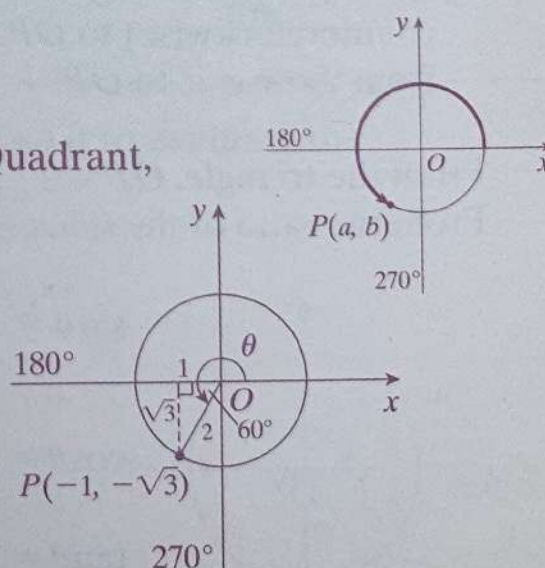
Since we are working in the 3rd Quadrant,

$$a = -1 \text{ and } b = -\sqrt{3}$$

$$\text{Therefore, } \sin \theta = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$



M 13 b.

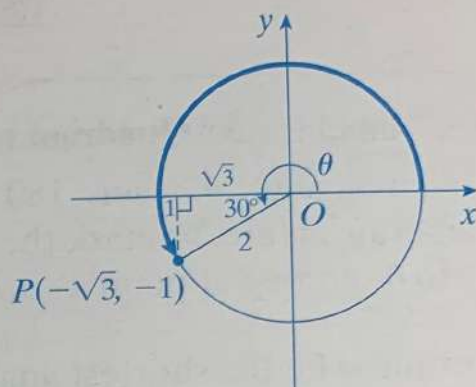
1. In each exercise, draw the diagram, and use it to evaluate the trigonometric functions of the given angle.

(1) $\theta = 210^\circ$

$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$

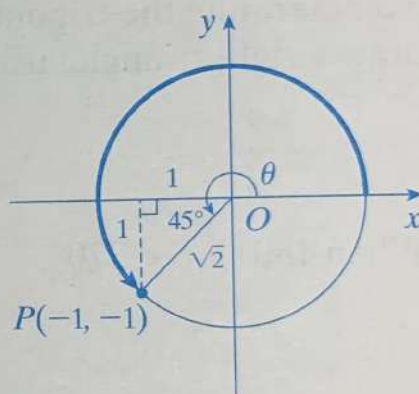


(2) $\theta = 225^\circ$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$



2. Given point $P(-1, -\sqrt{3})$, find the value of the sin, cos and tan of angle θ , formed from the positive x -axis (moving counterclockwise) to OP .

[Sol] Letting θ be the angle formed from the positive x -axis (moving counterclockwise) to OP , and letting α be the shortest angle formed from the x -axis to OP ,

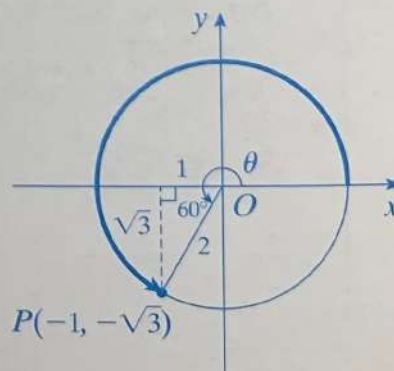
From the triangle, $OP = 2$

From the ratio of the sides, $\alpha = 60^\circ$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$



Trigonometric Functions 2

Time : to : Date Name

100%	90%	80%	70%	60%~
(mistakes) 0	-	-	1	2~

We can label the **4th Quadrant** to be from 270° to 360° .

Given an angle θ , where $270^\circ < \theta < 360^\circ$, we can draw an arc, to mark the measure from 0° to θ° ending at point $P(a, b)$.

Letting α be the shortest angle formed from the x -axis to OP ,

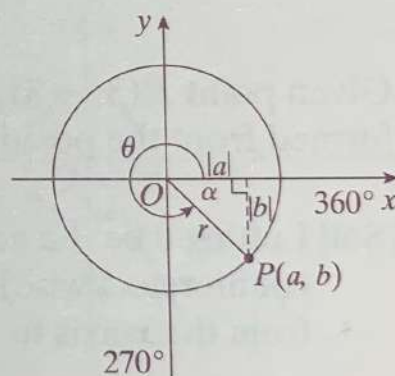
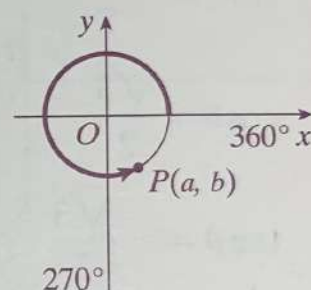
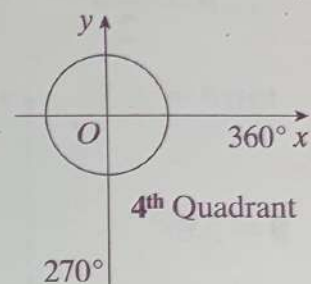
To determine the trigonometric values of θ , we draw a right triangle, using angle α .

$$\sin \theta = -\sin \alpha = \frac{b}{r}$$

When $270^\circ < \theta < 360^\circ$, $\cos \theta = \cos \alpha = \frac{a}{r}$

$$\tan \theta = -\tan \alpha = \frac{b}{a}$$

*Note that in the 4th Quadrant, $a > 0$ and $b < 0$.



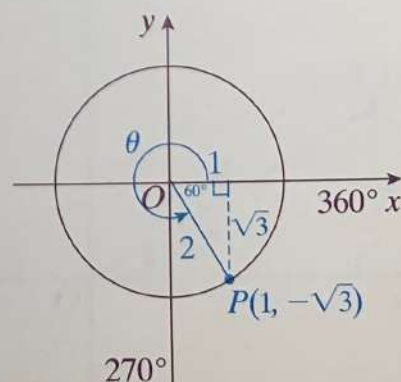
- In each exercise, draw the diagram, and use it to evaluate the trigonometric functions of the given angle.

(1) $\theta = 300^\circ$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$



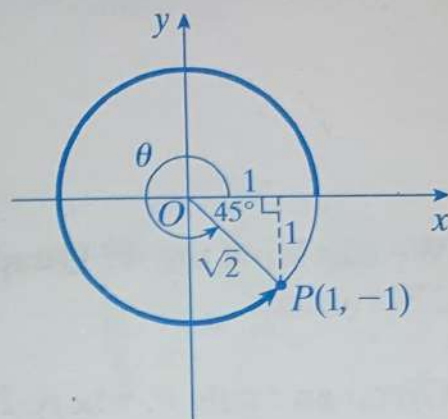
M 14 b

(2) $\theta = 315^\circ$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$

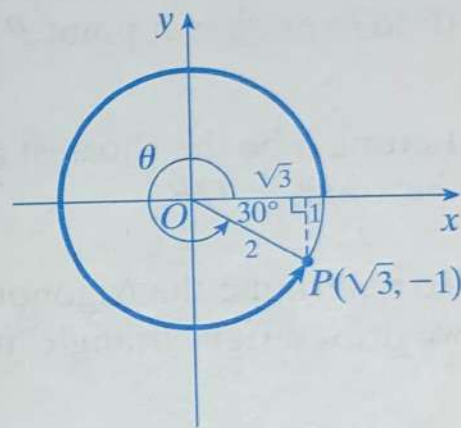


(3) $\theta = 330^\circ$

$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$



2. Given point $P(3, -3)$, find the value of the sin, cos and tan of angle θ , formed from the positive x -axis (moving counterclockwise) to OP .

[Sol] Letting θ be the angle formed from the positive x -axis (moving counterclockwise) to OP , and letting α be the shortest angle formed from the x -axis to OP ,

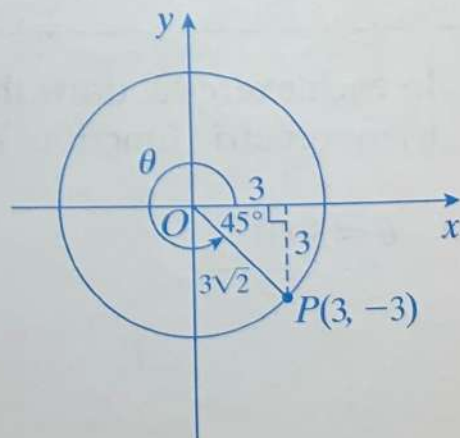
From the triangle, $OP = 3\sqrt{2}$

From the ratio of the sides, $\alpha = 45^\circ$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$



M 15 a

Trigonometric Functions 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	1	-	2~

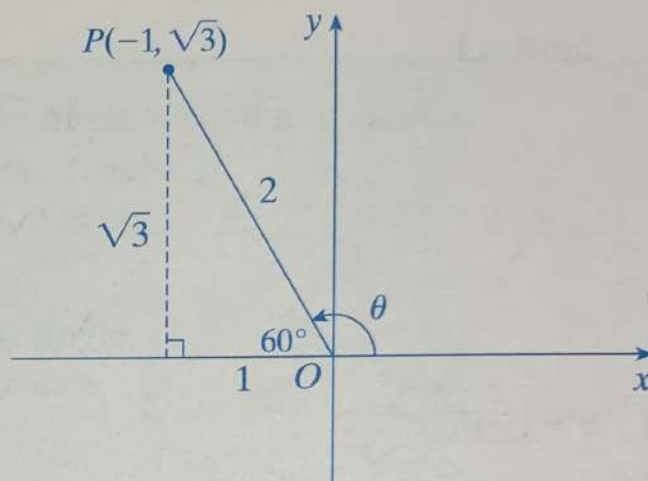
In each exercise, draw the diagram, and use it to evaluate the sin, cos and tan functions of the given angle.

(1) $\theta = 120^\circ$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$

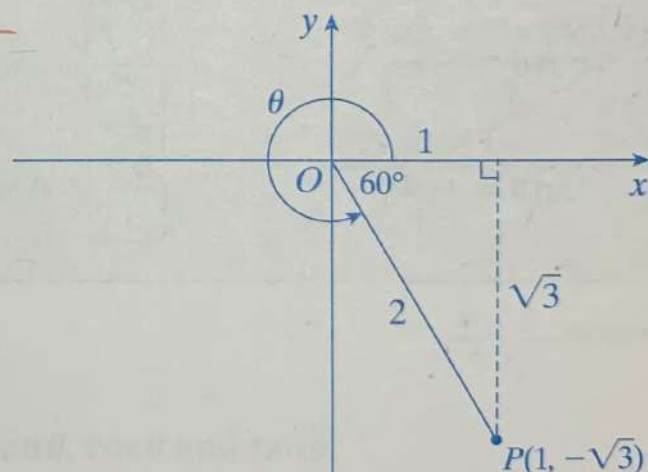


(2) $\theta = 300^\circ$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$

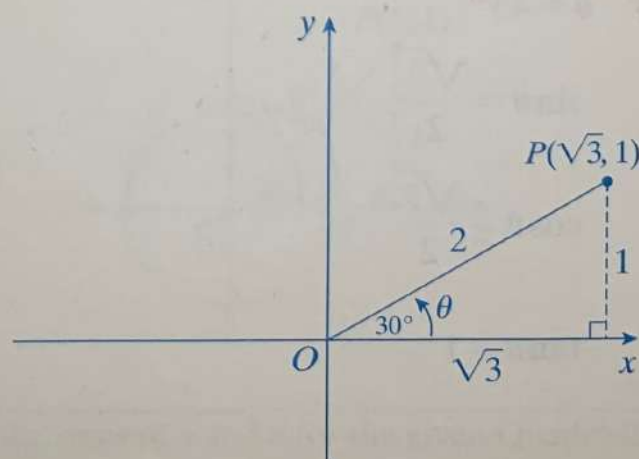


(3) $\theta = 30^\circ$

$$\sin \theta = \frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$



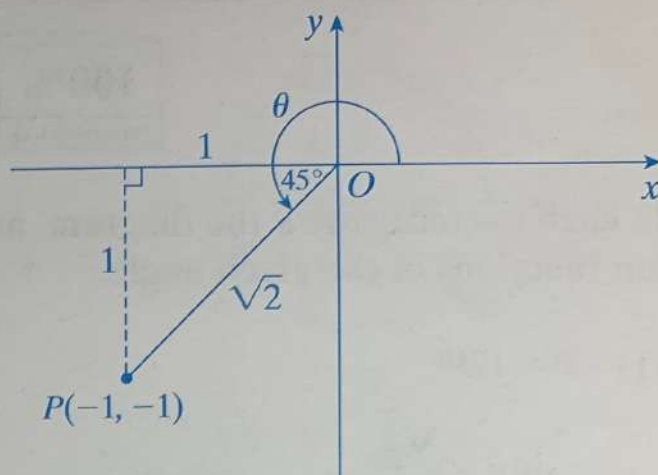
M 15 b

(4) $\theta = 225^\circ$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$

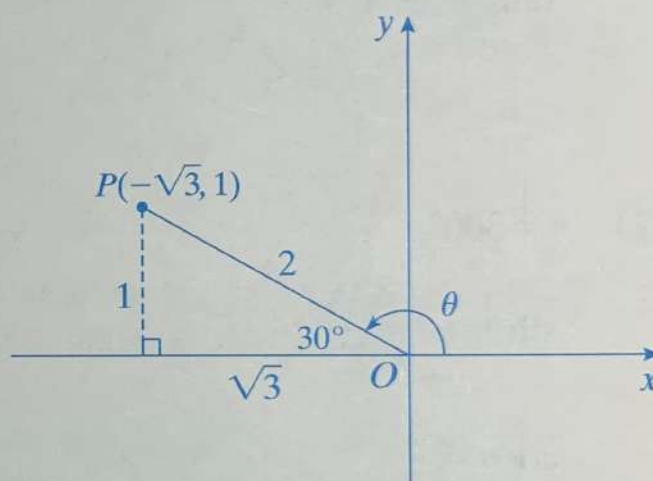


(5) $\theta = 150^\circ$

$$\sin \theta = \frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$

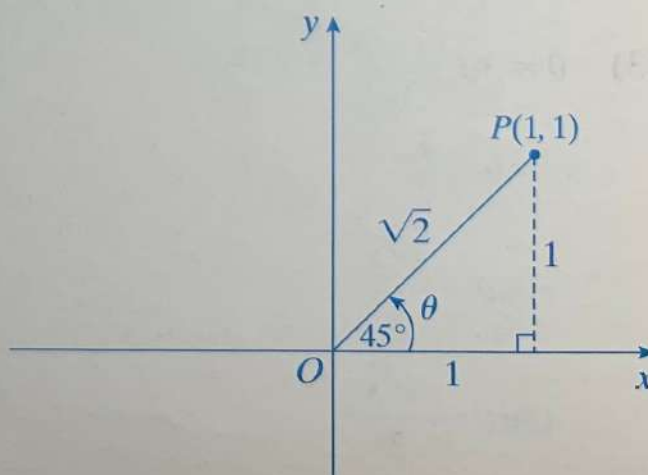


(6) $\theta = 45^\circ$

$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$



Trigonometric Functions 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	-	1	2~

Once we move one complete revolution, i.e. 360° around the circle, we can use the method studied in M11-15 to evaluate the trigonometric functions of angles greater than 360° .

Ex.

Given $\theta = 390^\circ$, find the value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

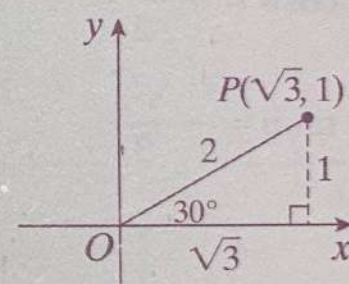
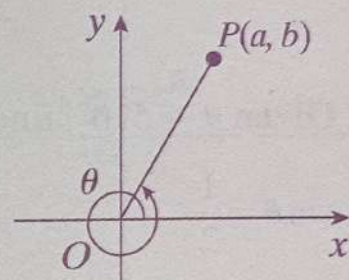
[Sol] As we measure θ , once we reach 360° we move 30° more, placing P in the 1st Quadrant.

Letting α be the shortest angle formed by OP and the x -axis,
 $\alpha = 30^\circ$

From the graph, $\sin\theta = \frac{1}{2}$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{\sqrt{3}}{3}$$



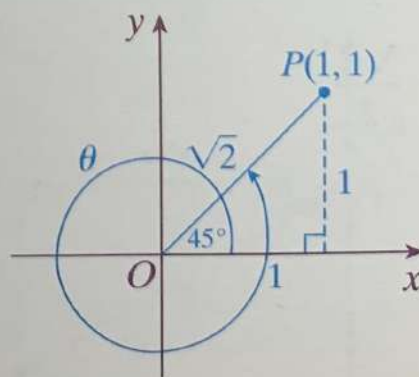
Answers: $\frac{2}{\sqrt{3}}, \frac{3}{\sqrt{3}}$

1. Given $\theta = 405^\circ$, find the value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

$$\sin\theta = \frac{\sqrt{2}}{2}$$

$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\tan\theta = 1$$



Reminder: In each exercise always check the signs of a and b for the given Quadrant.

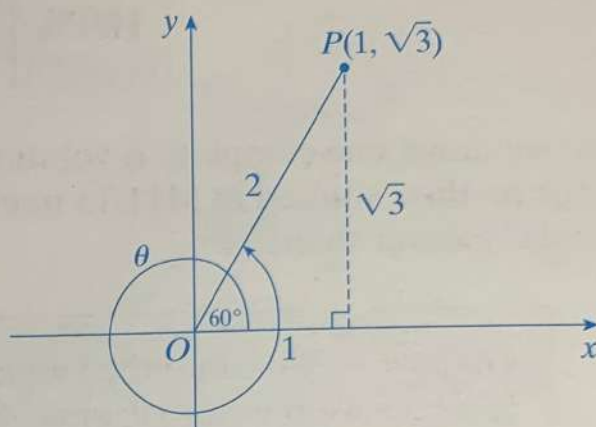
M 16 b

2. Given $\theta = 420^\circ$, find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$

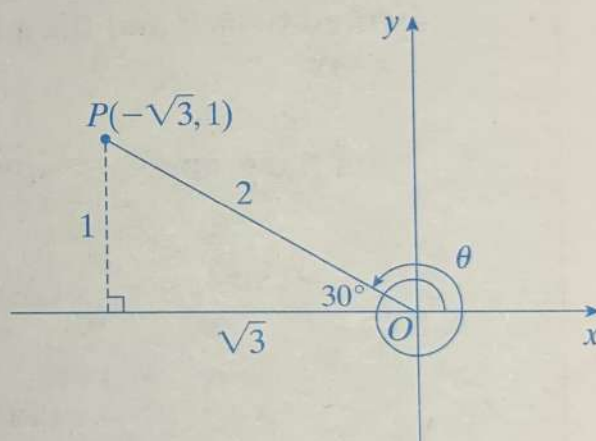


3. Given $\theta = 510^\circ$, find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

$$\sin \theta = \frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$

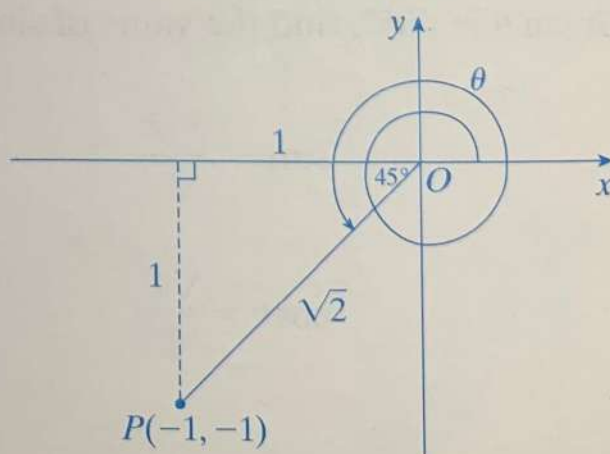


4. Given $\theta = 585^\circ$, find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$



M 17 a

Trigonometric Functions 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	1	-	2~

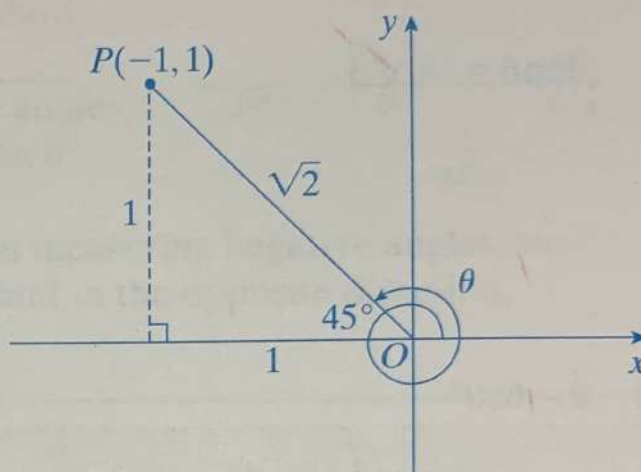
For each given angle θ , find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

(1) $\theta = 495^\circ$

$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$

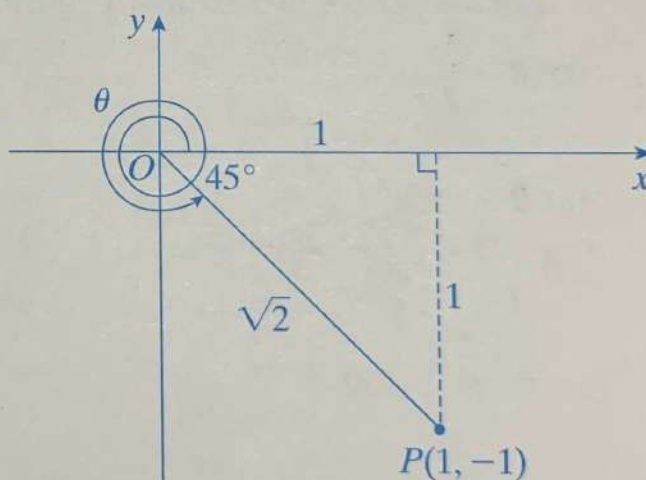


(2) $\theta = 675^\circ$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$

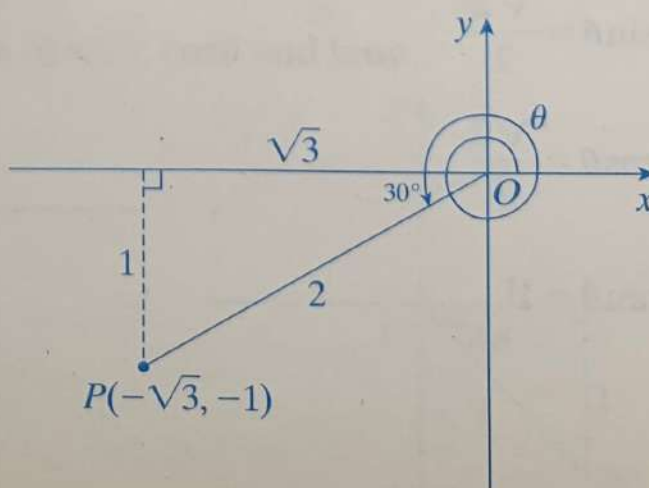


(3) $\theta = 570^\circ$

$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$



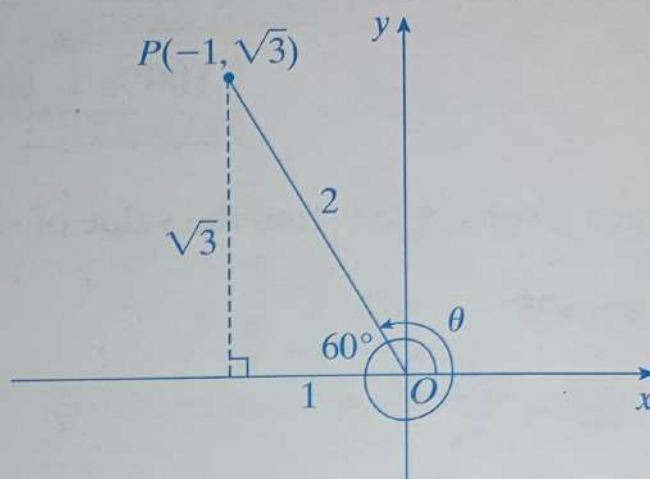
M 17 b

(4) $\theta = 480^\circ$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$

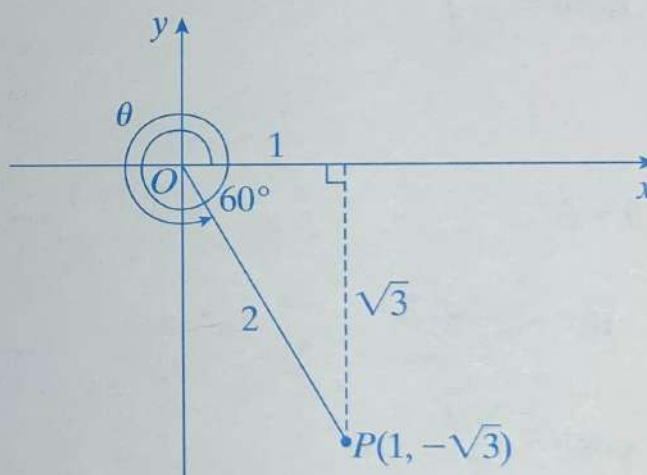


(5) $\theta = 660^\circ$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$

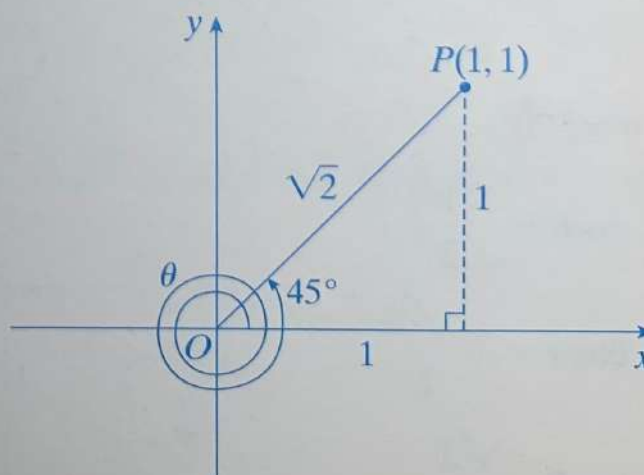


(6)* $\theta = 765^\circ$

$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$



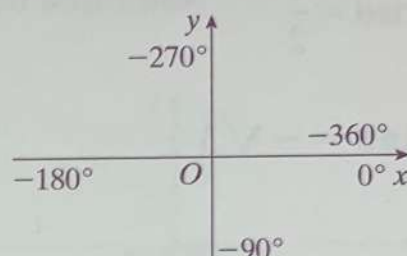
M 18 a

Trigonometric Functions 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	-	1	2~

Up to this point we have been working with positive angles. We can use the method studied in M11-17 to evaluate the trigonometric functions of negative angles, i.e. angles whose measure is less than 0° .



As shown on the figure above, when measuring negative angles, we measure the degrees of each Quadrant in the opposite direction, clockwise.

Ex.

Given $\theta = -30^\circ$, find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

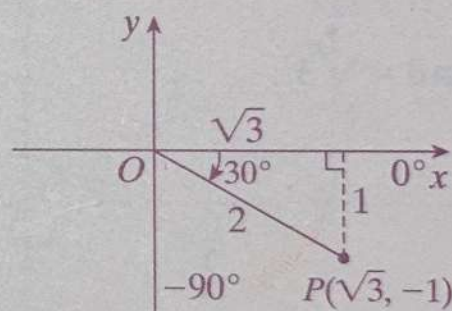
[Sol] We begin measuring from 0° towards the 4th Quadrant.

From the graph,

$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$



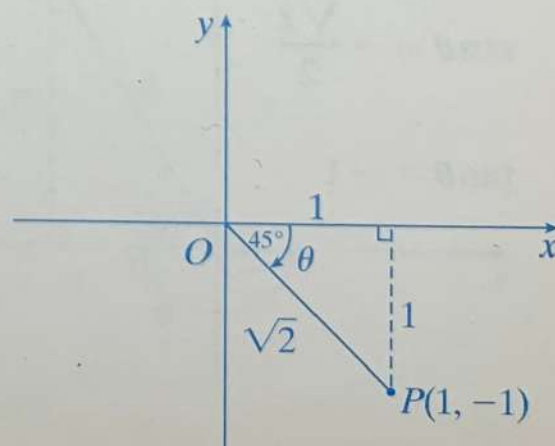
Answers: $\frac{\sqrt{3}}{2}$, $-\frac{1}{2}$, $-\frac{\sqrt{3}}{3}$

1. Given $\theta = -45^\circ$, find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$



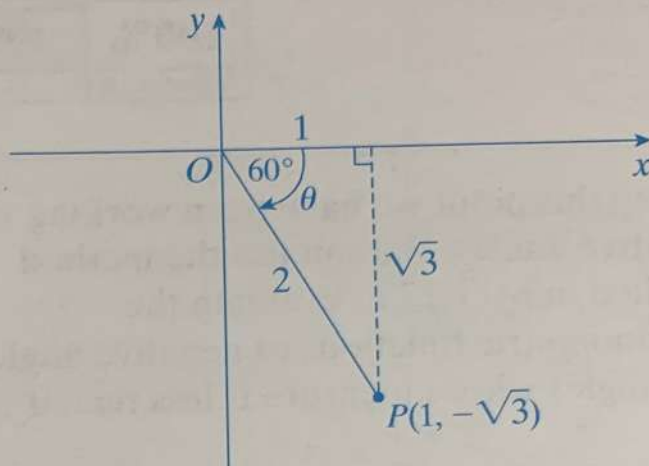
M 18 b

2. Given $\theta = -60^\circ$, find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$

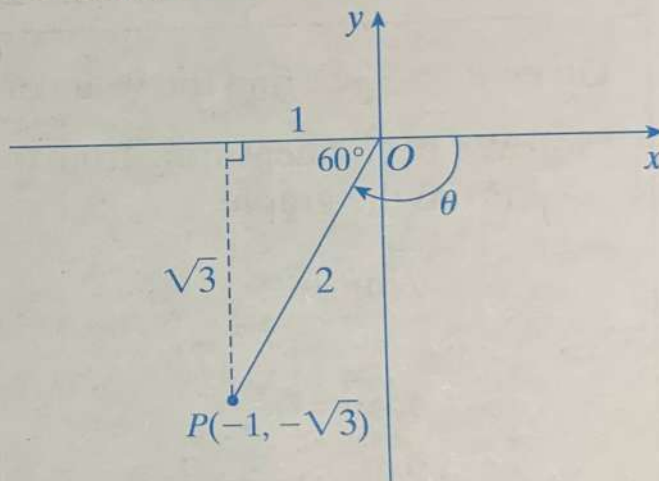


3. Given $\theta = -120^\circ$, find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$

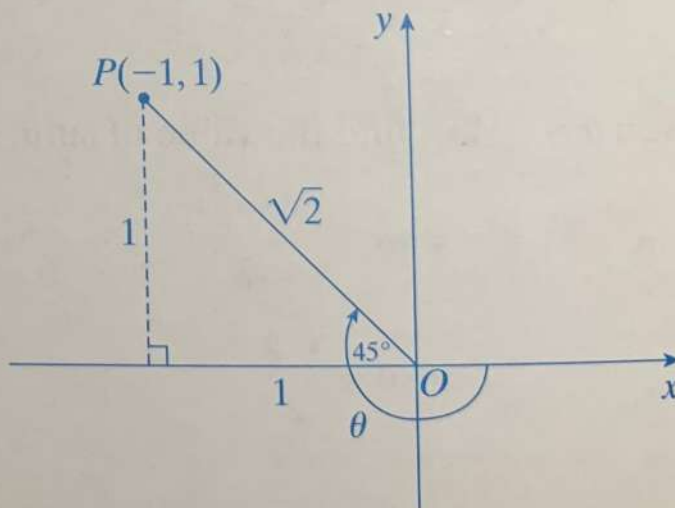


4. Given $\theta = -225^\circ$, find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$



Trigonometric Functions 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	1	-	2~

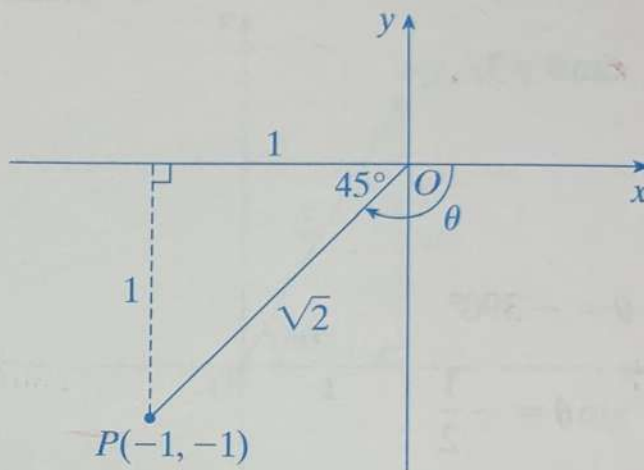
For each given angle θ find the value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

(1) $\theta = -135^\circ$

$$\sin\theta = -\frac{\sqrt{2}}{2}$$

$$\cos\theta = -\frac{\sqrt{2}}{2}$$

$$\tan\theta = 1$$

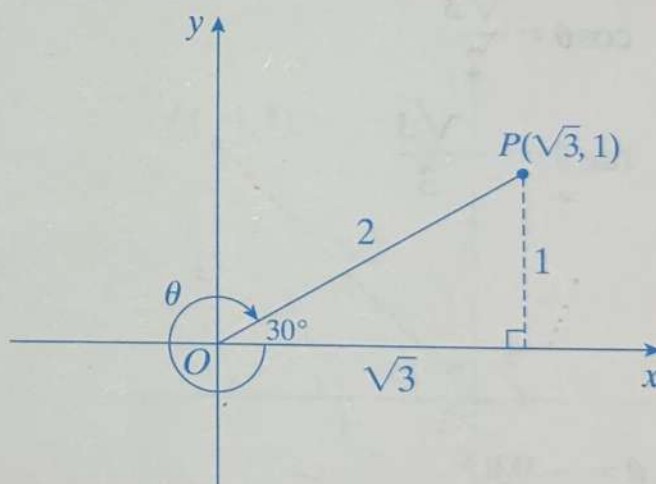


(2) $\theta = -330^\circ$

$$\sin\theta = \frac{1}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{\sqrt{3}}{3}$$

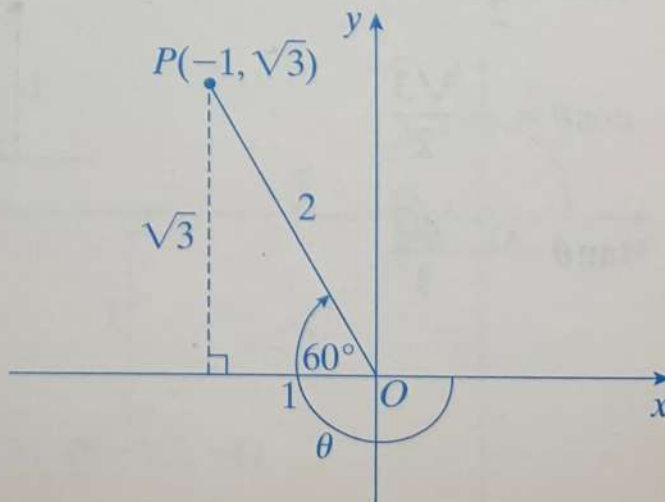


(3) $\theta = -240^\circ$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\cos\theta = -\frac{1}{2}$$

$$\tan\theta = -\sqrt{3}$$



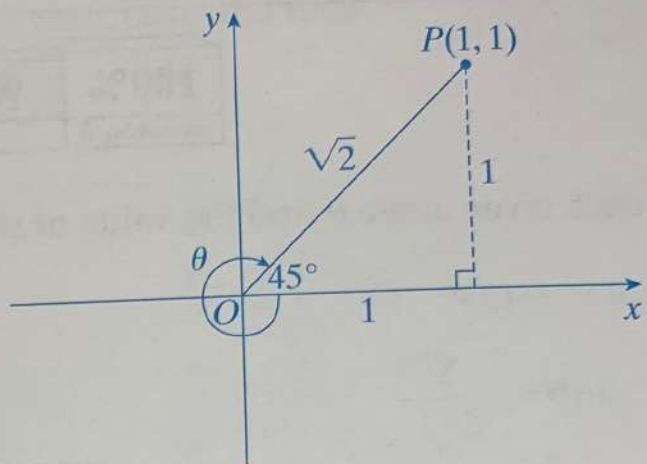
M 19 b

(4) $\theta = -315^\circ$

$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$

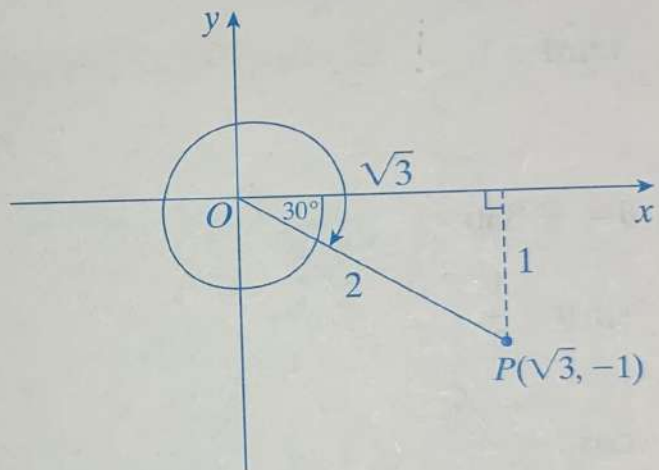


(5) $\theta = -390^\circ$

$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$

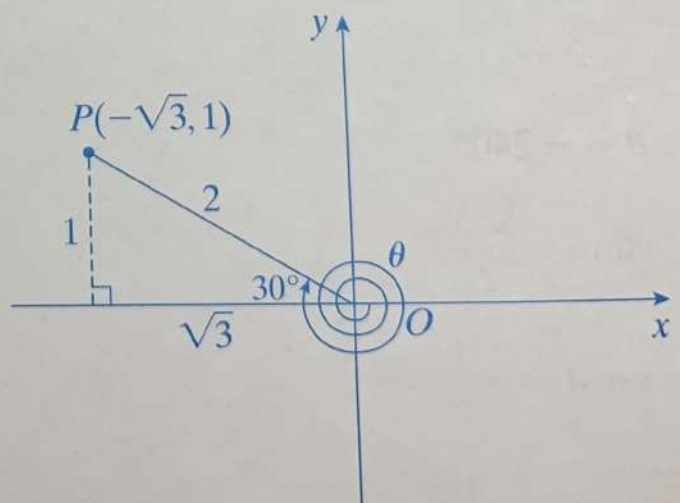


(6)* $\theta = -930^\circ$

$$\sin \theta = \frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$



Trigonometric Functions 2

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	1	-	2~

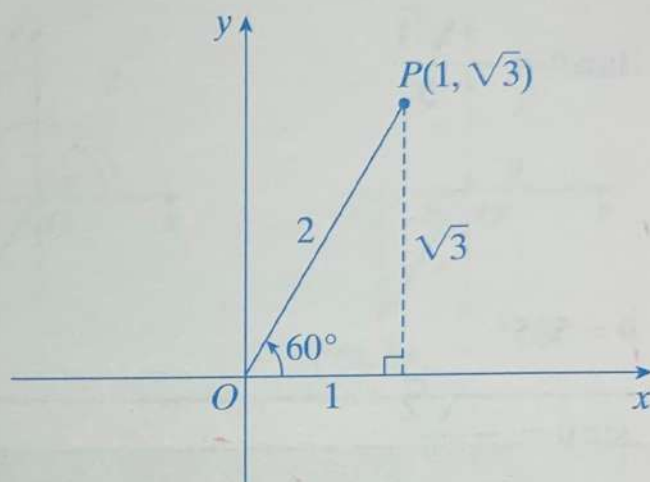
For each given angle θ find the value of $\sin \theta$, $\cos \theta$ and $\tan \theta$.

(1) $\theta = 60^\circ$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$

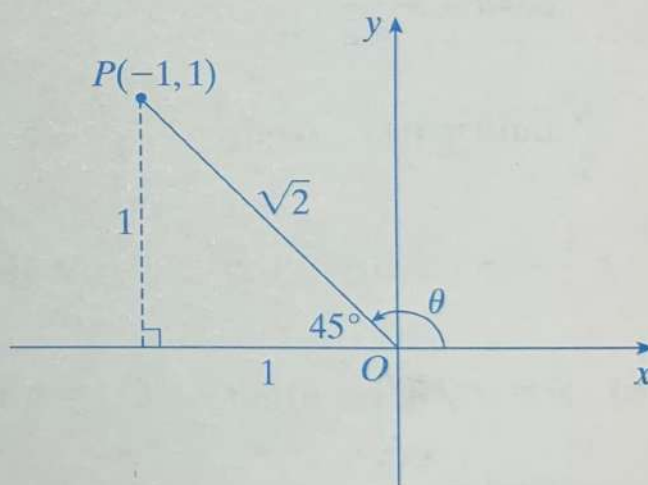


(2) $\theta = 135^\circ$

$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = -1$$

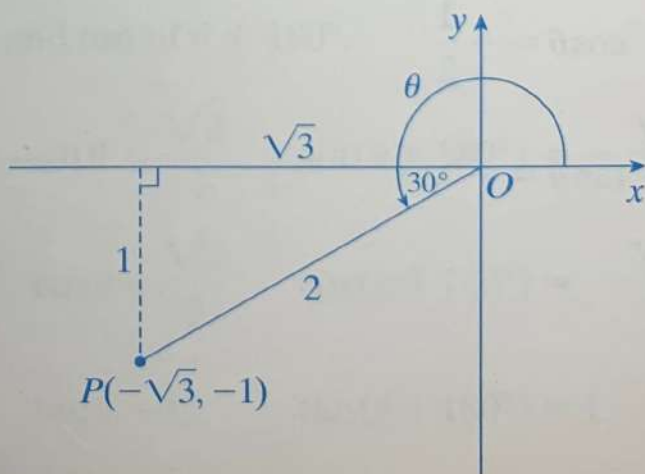


(3) $\theta = 210^\circ$

$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$



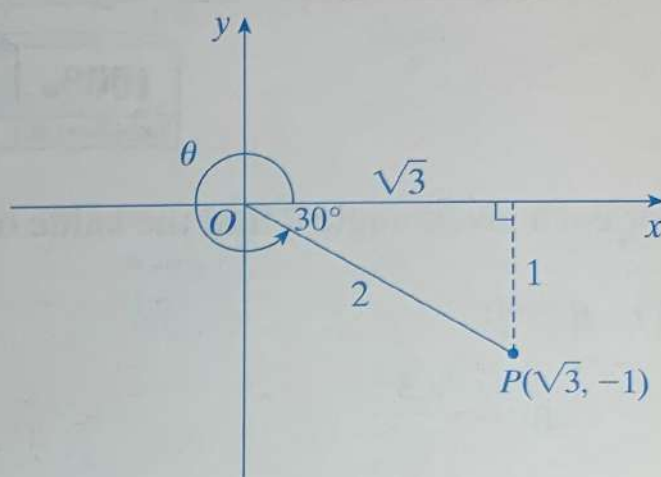
M 20 b

(4) $\theta = 330^\circ$

$$\sin \theta = -\frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$

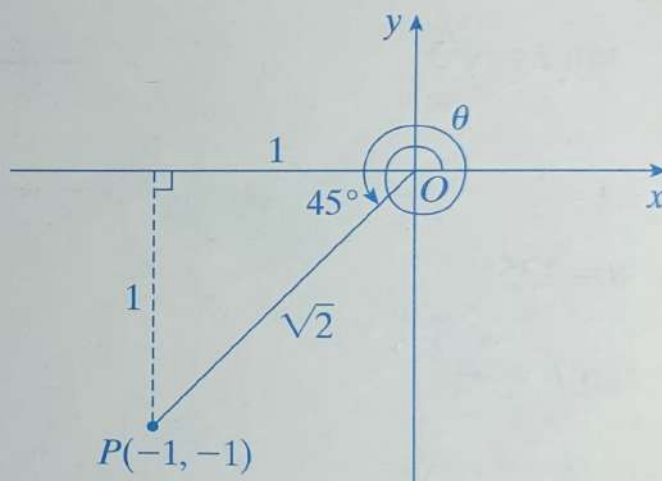


(5) $\theta = 585^\circ$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$

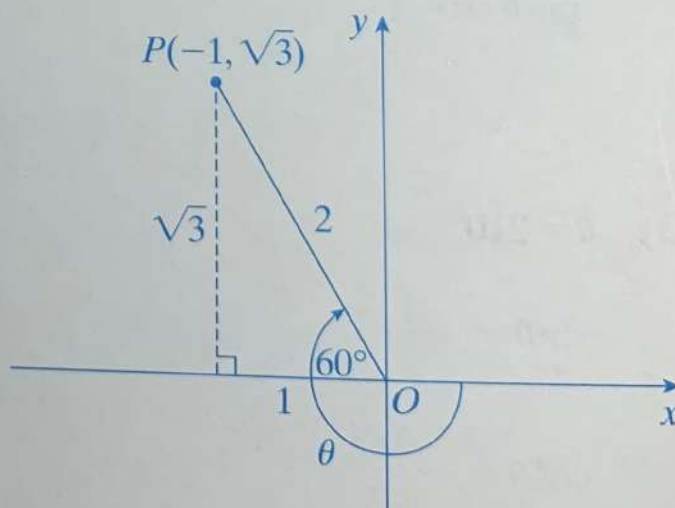


(6) $\theta = -240^\circ$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$

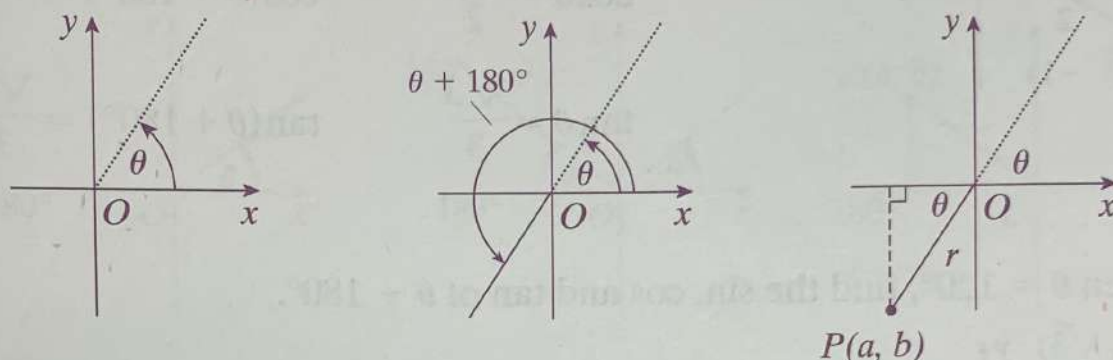


Trigonometric Functions 3

Time : to : Date Name

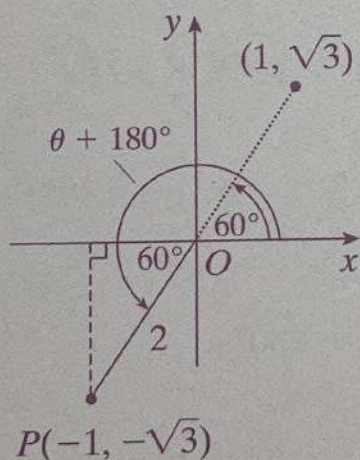
100%	90%	80%	70%	69%~
(mistakes) 0	-	-	1	2~

From θ , measuring 180° counterclockwise results to $\theta + 180^\circ$.



Ex.

Given $\theta = 60^\circ$, find the sin, cos and tan of $\theta + 180^\circ$.

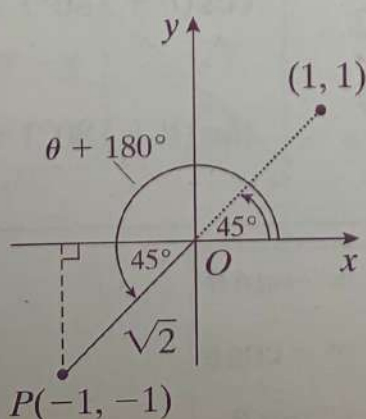


$$\sin \theta = \frac{\sqrt{3}}{2} \quad \sin(\theta + 180^\circ) = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2} \quad \cos(\theta + 180^\circ) = -\frac{1}{2}$$

$$\tan \theta = \sqrt{3} \quad \tan(\theta + 180^\circ) = \sqrt{3}$$

1. Given $\theta = 45^\circ$, find the sin, cos and tan of $\theta + 180^\circ$.



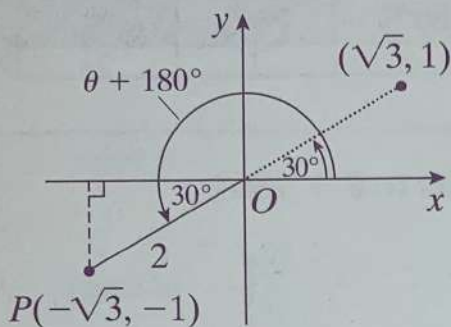
$$\sin \theta = \frac{\sqrt{2}}{2} \quad \sin(\theta + 180^\circ) = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2} \quad \cos(\theta + 180^\circ) = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = 1 \quad \tan(\theta + 180^\circ) = 1$$

M 21 b

2. Given $\theta = 30^\circ$, find the sin, cos and tan of $\theta + 180^\circ$.



$$\sin \theta = \frac{1}{2}$$

$$\sin(\theta + 180^\circ) = -\frac{1}{2}$$

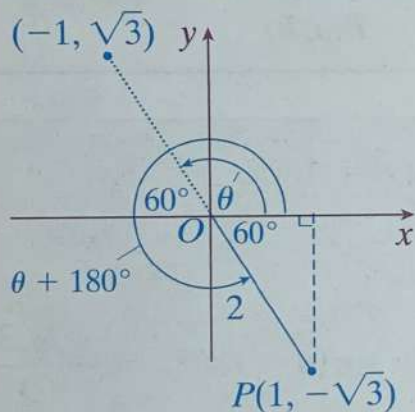
$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\cos(\theta + 180^\circ) = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$

$$\tan(\theta + 180^\circ) = \frac{\sqrt{3}}{3}$$

3. Given $\theta = 120^\circ$, find the sin, cos and tan of $\theta + 180^\circ$.



$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\sin(\theta + 180^\circ) = -\frac{\sqrt{3}}{2}$$

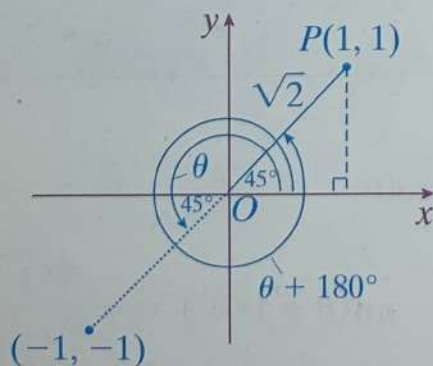
$$\cos \theta = -\frac{1}{2}$$

$$\cos(\theta + 180^\circ) = \frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$

$$\tan(\theta + 180^\circ) = -\sqrt{3}$$

4. Given $\theta = 225^\circ$, find the sin, cos and tan of $\theta + 180^\circ$.



$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\sin(\theta + 180^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos \theta = -\frac{\sqrt{2}}{2}$$

$$\cos(\theta + 180^\circ) = \frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$

$$\tan(\theta + 180^\circ) = 1$$

Formulas

$$\sin(\theta + 180^\circ) = -\sin \theta$$

$$\cos(\theta + 180^\circ) = -\cos \theta$$

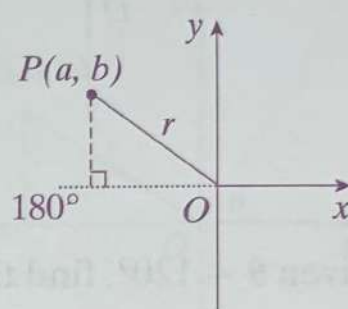
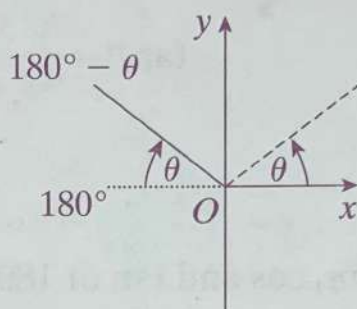
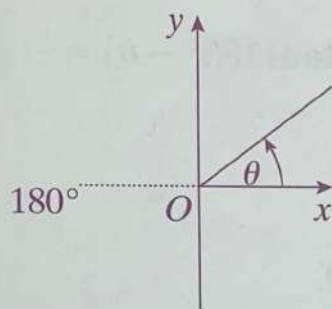
$$\tan(\theta + 180^\circ) = \tan \theta$$

Trigonometric Functions 3

Time : to : Date Name

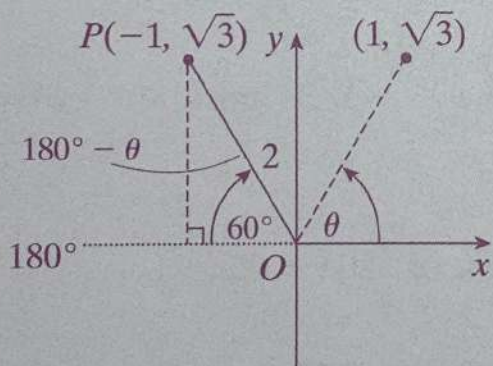
100%	90%	80%	70%	69%~
(mistakes) 0	-	-	1	2~

From 180° , measuring θ degrees clockwise results to $180^\circ - \theta$.



Ex.

Given $\theta = 60^\circ$, find the sin, cos and tan of $180^\circ - \theta$.



$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\sin(180^\circ - \theta) = \frac{\sqrt{3}}{2}$$

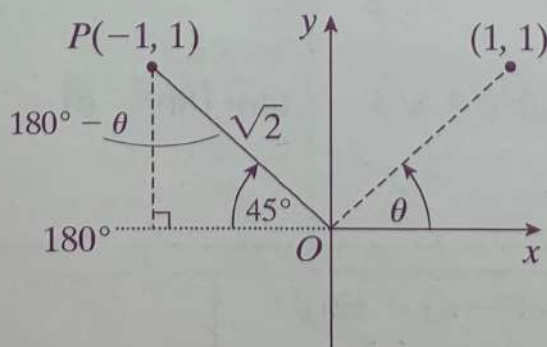
$$\cos \theta = \frac{1}{2}$$

$$\cos(180^\circ - \theta) = -\frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$

$$\tan(180^\circ - \theta) = -\sqrt{3}$$

1. Given $\theta = 45^\circ$, find the sin, cos and tan of $180^\circ - \theta$.



$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\sin(180^\circ - \theta) = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

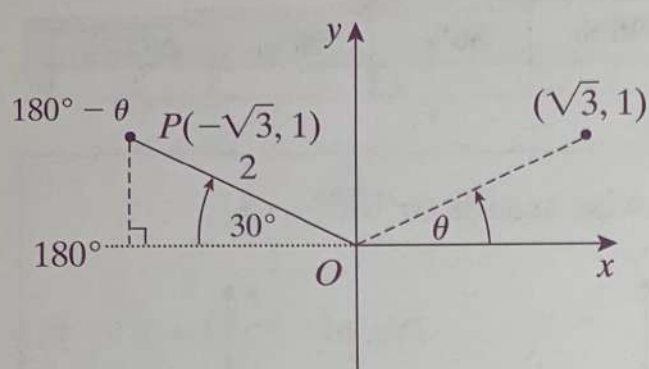
$$\cos(180^\circ - \theta) = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$

$$\tan(180^\circ - \theta) = -1$$

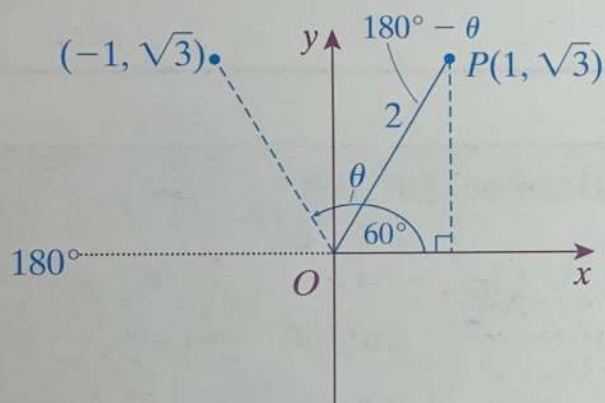
M 22 b

2. Given $\theta = 30^\circ$, find the sin, cos and tan of $180^\circ - \theta$.



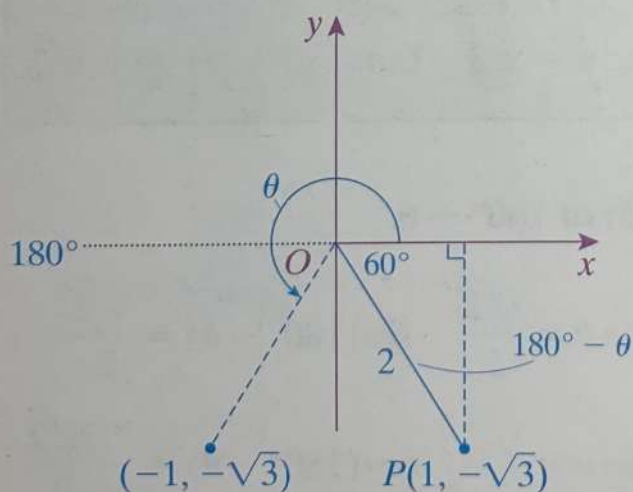
$$\begin{aligned}\sin \theta &= \frac{1}{2} & \sin(180^\circ - \theta) &= \frac{1}{2} \\ \cos \theta &= \frac{\sqrt{3}}{2} & \cos(180^\circ - \theta) &= -\frac{\sqrt{3}}{2} \\ \tan \theta &= \frac{\sqrt{3}}{3} & \tan(180^\circ - \theta) &= -\frac{\sqrt{3}}{3}\end{aligned}$$

3. Given $\theta = 120^\circ$, find the sin, cos and tan of $180^\circ - \theta$.



$$\begin{aligned}\sin \theta &= \frac{\sqrt{3}}{2} & \sin(180^\circ - \theta) &= \frac{\sqrt{3}}{2} \\ \cos \theta &= -\frac{1}{2} & \cos(180^\circ - \theta) &= \frac{1}{2} \\ \tan \theta &= -\sqrt{3} & \tan(180^\circ - \theta) &= \sqrt{3}\end{aligned}$$

4. Given $\theta = 240^\circ$, find the sin, cos and tan of $180^\circ - \theta$.



$$\begin{aligned}\sin \theta &= -\frac{\sqrt{3}}{2} & \sin(180^\circ - \theta) &= -\frac{\sqrt{3}}{2} \\ \cos \theta &= -\frac{1}{2} & \cos(180^\circ - \theta) &= \frac{1}{2} \\ \tan \theta &= \sqrt{3} & \tan(180^\circ - \theta) &= -\sqrt{3}\end{aligned}$$

Formulas

$$\sin(180^\circ - \theta) = \sin \theta$$

$$\cos(180^\circ - \theta) = -\cos \theta$$

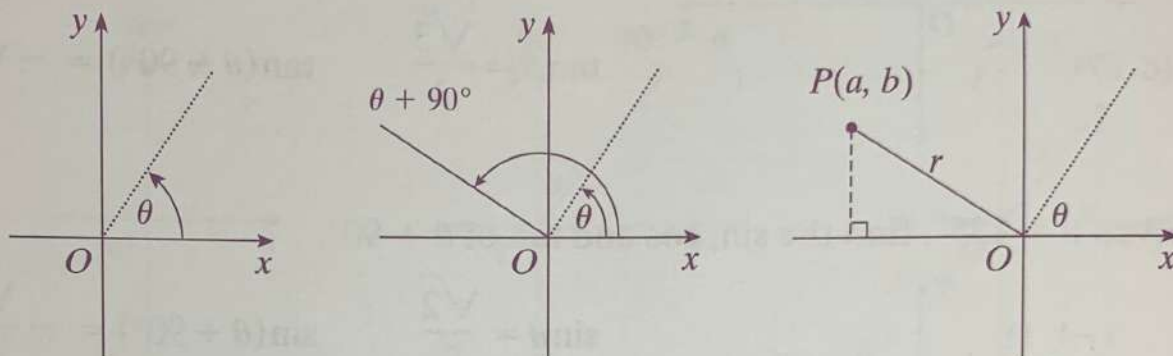
$$\tan(180^\circ - \theta) = -\tan \theta$$

Trigonometric Functions 3

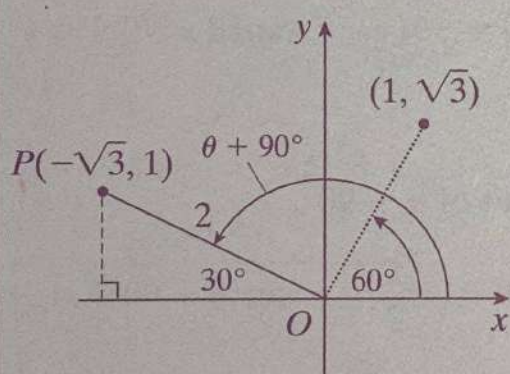
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From θ , measuring 90° counterclockwise results to $\theta + 90^\circ$.



Ex. Given $\theta = 60^\circ$, find the sin, cos and tan of $\theta + 90^\circ$.

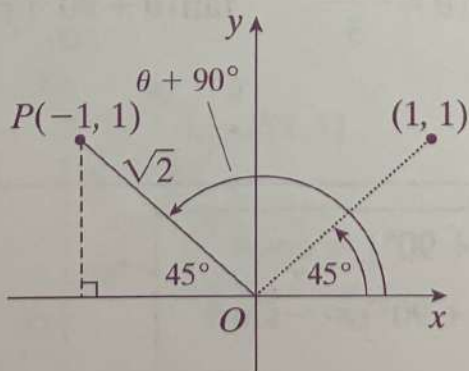


$$\sin \theta = \frac{\sqrt{3}}{2} \quad \sin(\theta + 90^\circ) = \frac{1}{2}$$

$$\cos \theta = \frac{1}{2} \quad \cos(\theta + 90^\circ) = -\frac{\sqrt{3}}{2}$$

$$\tan \theta = \sqrt{3} \quad \tan(\theta + 90^\circ) = -\frac{\sqrt{3}}{3}$$

1. Given $\theta = 45^\circ$, find the sin, cos and tan of $\theta + 90^\circ$.



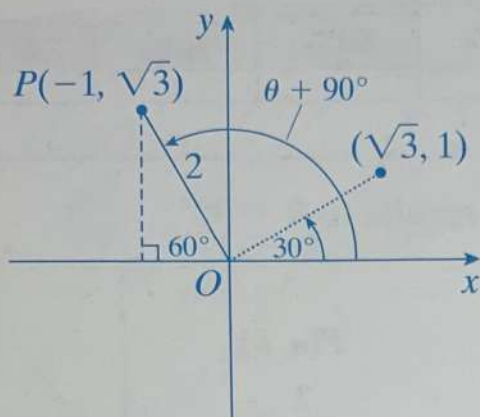
$$\sin \theta = \frac{\sqrt{2}}{2} \quad \sin(\theta + 90^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2} \quad \cos(\theta + 90^\circ) = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = 1 \quad \tan(\theta + 90^\circ) = -1$$

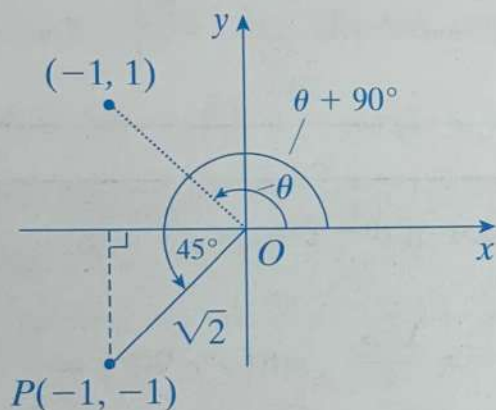
M 23 b

2. Given $\theta = 30^\circ$, find the sin, cos and tan of $\theta + 90^\circ$.



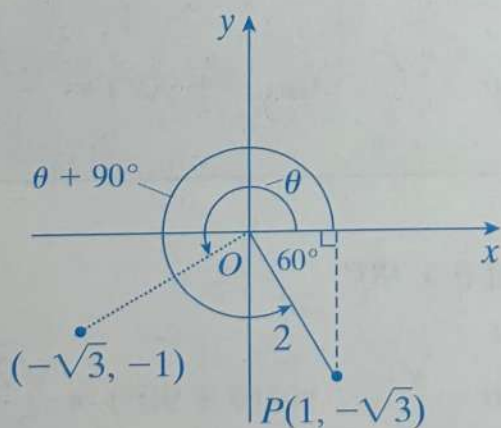
$$\begin{aligned}\sin \theta &= \frac{1}{2} & \sin(\theta + 90^\circ) &= \frac{\sqrt{3}}{2} \\ \cos \theta &= \frac{\sqrt{3}}{2} & \cos(\theta + 90^\circ) &= -\frac{1}{2} \\ \tan \theta &= \frac{\sqrt{3}}{3} & \tan(\theta + 90^\circ) &= -\sqrt{3}\end{aligned}$$

3. Given $\theta = 135^\circ$, find the sin, cos and tan of $\theta + 90^\circ$.



$$\begin{aligned}\sin \theta &= \frac{\sqrt{2}}{2} & \sin(\theta + 90^\circ) &= -\frac{\sqrt{2}}{2} \\ \cos \theta &= -\frac{\sqrt{2}}{2} & \cos(\theta + 90^\circ) &= -\frac{\sqrt{2}}{2} \\ \tan \theta &= -1 & \tan(\theta + 90^\circ) &= 1\end{aligned}$$

4. Given $\theta = 210^\circ$, find the sin, cos and tan of $\theta + 90^\circ$.



$$\begin{aligned}\sin \theta &= -\frac{1}{2} & \sin(\theta + 90^\circ) &= -\frac{\sqrt{3}}{2} \\ \cos \theta &= -\frac{\sqrt{3}}{2} & \cos(\theta + 90^\circ) &= \frac{1}{2} \\ \tan \theta &= \frac{\sqrt{3}}{3} & \tan(\theta + 90^\circ) &= -\sqrt{3}\end{aligned}$$

Formulas

$$\sin(\theta + 90^\circ) = \cos \theta$$

$$\cos(\theta + 90^\circ) = -\sin \theta$$

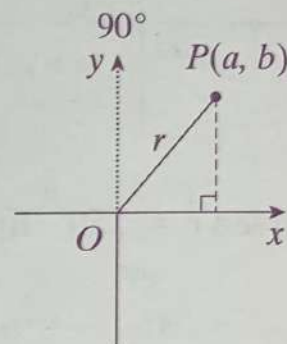
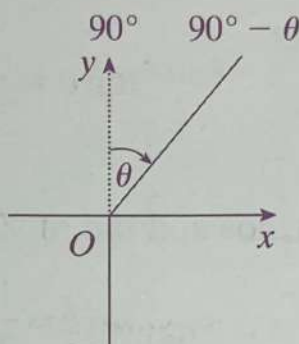
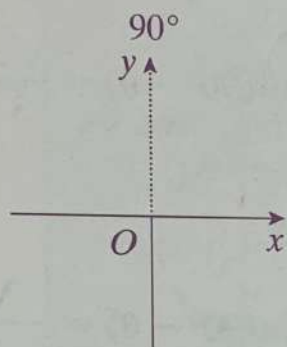
$$\tan(\theta + 90^\circ) = -\frac{1}{\tan \theta}$$

Trigonometric Functions 3

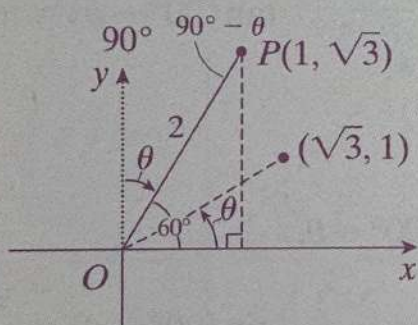
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From 90° , measuring θ degrees clockwise results to $90^\circ - \theta$.



Ex. Given $\theta = 30^\circ$, find the sin, cos and tan of $90^\circ - \theta$.



$$\sin \theta = \frac{1}{2}$$

$$\sin(90^\circ - \theta) = \frac{\sqrt{3}}{2}$$

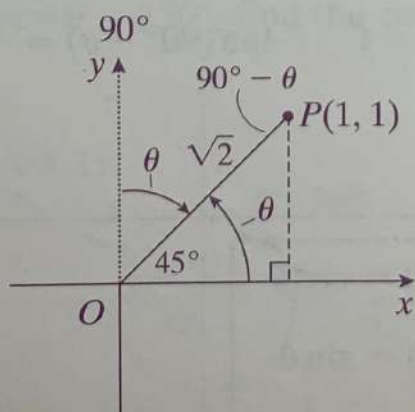
$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\cos(90^\circ - \theta) = \frac{1}{2}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$

$$\tan(90^\circ - \theta) = \sqrt{3}$$

1. Given $\theta = 45^\circ$, find the sin, cos and tan of $90^\circ - \theta$.



$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\sin(90^\circ - \theta) = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

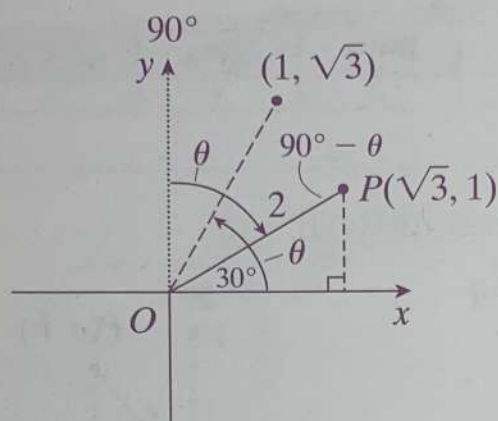
$$\cos(90^\circ - \theta) = \frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$

$$\tan(90^\circ - \theta) = 1$$

M 24 b

2. Given $\theta = 60^\circ$, find the sin, cos and tan of $90^\circ - \theta$.

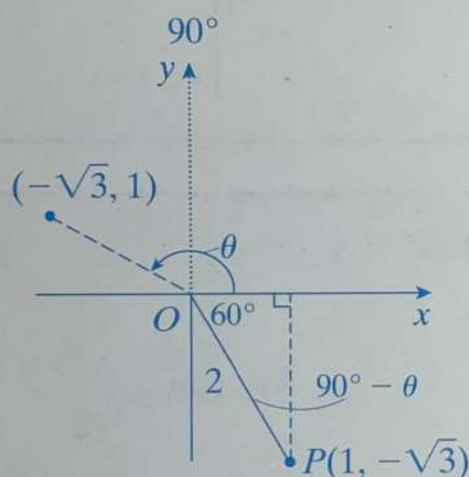


$$\sin \theta = \frac{\sqrt{3}}{2} \quad \sin(90^\circ - \theta) = \frac{1}{2}$$

$$\cos \theta = \frac{1}{2} \quad \cos(90^\circ - \theta) = \frac{\sqrt{3}}{2}$$

$$\tan \theta = \sqrt{3} \quad \tan(90^\circ - \theta) = \frac{\sqrt{3}}{3}$$

3. Given $\theta = 150^\circ$, find the sin, cos and tan of $90^\circ - \theta$.

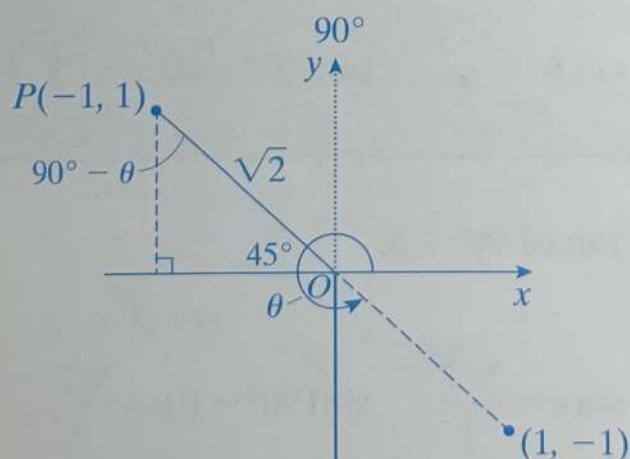


$$\sin \theta = \frac{1}{2} \quad \sin(90^\circ - \theta) = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2} \quad \cos(90^\circ - \theta) = \frac{1}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3} \quad \tan(90^\circ - \theta) = -\sqrt{3}$$

4. Given $\theta = 315^\circ$, find the sin, cos and tan of $90^\circ - \theta$.



$$\sin \theta = -\frac{\sqrt{2}}{2} \quad \sin(90^\circ - \theta) = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2} \quad \cos(90^\circ - \theta) = -\frac{\sqrt{2}}{2}$$

$$\tan \theta = -1 \quad \tan(90^\circ - \theta) = -1$$

Formulas

$$\sin(90^\circ - \theta) = \cos \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\tan(90^\circ - \theta) = \frac{1}{\tan \theta}$$

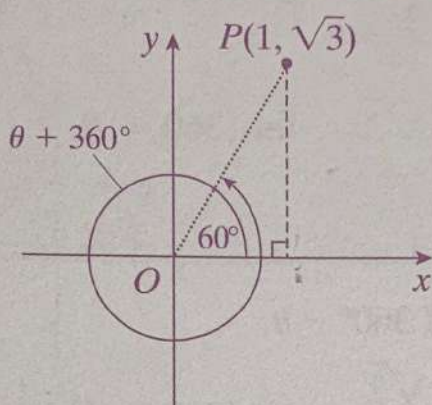
M 25 a

Trigonometric Functions 3

Time : to : Date Name

100%	90%	80%	70%	69%~
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Ex.

Given $\theta = 60^\circ$, find the sin, cos and tan of $\theta + 360^\circ$.

$$\sin \theta = \frac{\sqrt{3}}{2}$$

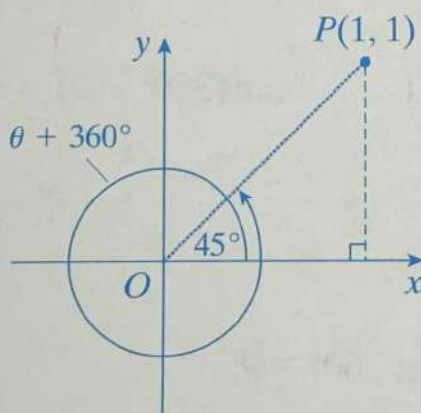
$$\sin(\theta + 360^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\cos(\theta + 360^\circ) = \frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$

$$\tan(\theta + 360^\circ) = \sqrt{3}$$

1. Given $\theta = 45^\circ$, find the sin, cos and tan of $\theta + 360^\circ$.

$$\sin \theta = \frac{\sqrt{2}}{2}$$

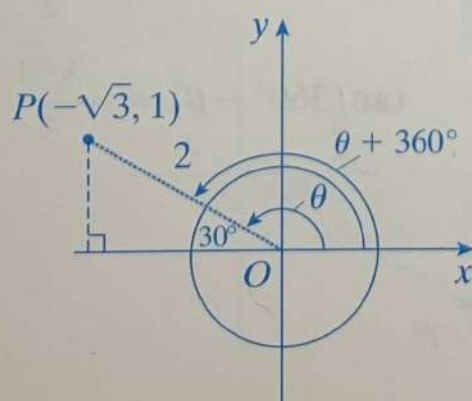
$$\sin(\theta + 360^\circ) = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\cos(\theta + 360^\circ) = \frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$

$$\tan(\theta + 360^\circ) = 1$$

2. Given $\theta = 150^\circ$, find the sin, cos and tan of $\theta + 360^\circ$.

$$\sin \theta = \frac{1}{2}$$

$$\sin(\theta + 360^\circ) = \frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2}$$

$$\cos(\theta + 360^\circ) = -\frac{\sqrt{3}}{2}$$

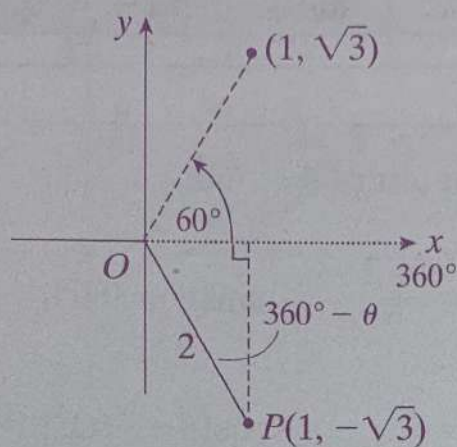
$$\tan \theta = -\frac{\sqrt{3}}{3}$$

$$\tan(\theta + 360^\circ) = -\frac{\sqrt{3}}{3}$$

M 25 b

Ex.

Given $\theta = 60^\circ$, find the sin, cos and tan of $360^\circ - \theta$.

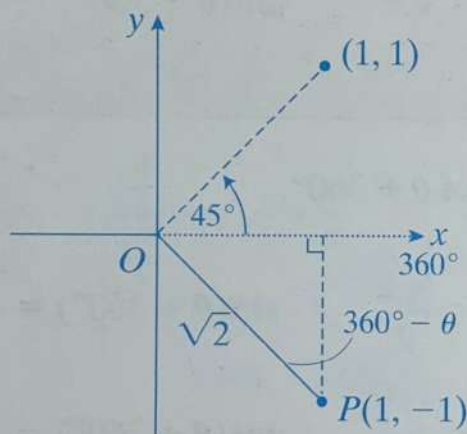


$$\sin \theta = \frac{\sqrt{3}}{2} \quad \sin(360^\circ - \theta) = -\frac{\sqrt{3}}{2}$$

$$\cos \theta = \frac{1}{2} \quad \cos(360^\circ - \theta) = \frac{1}{2}$$

$$\tan \theta = \sqrt{3} \quad \tan(360^\circ - \theta) = -\sqrt{3}$$

3. Given $\theta = 45^\circ$, find the sin, cos and tan of $360^\circ - \theta$.

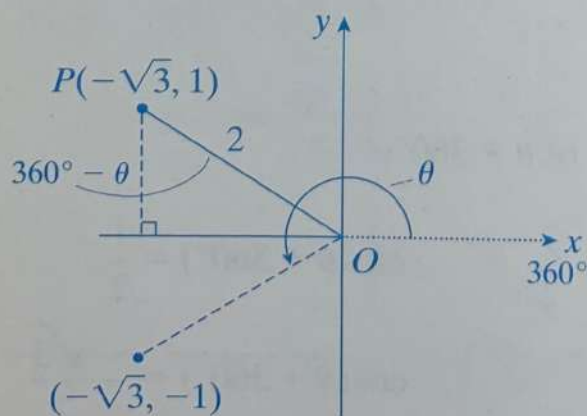


$$\sin \theta = \frac{\sqrt{2}}{2} \quad \sin(360^\circ - \theta) = -\frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2} \quad \cos(360^\circ - \theta) = \frac{\sqrt{2}}{2}$$

$$\tan \theta = 1 \quad \tan(360^\circ - \theta) = -1$$

4. Given $\theta = 210^\circ$, find the sin, cos and tan of $360^\circ - \theta$.



$$\sin \theta = -\frac{1}{2} \quad \sin(360^\circ - \theta) = \frac{1}{2}$$

$$\cos \theta = -\frac{\sqrt{3}}{2} \quad \cos(360^\circ - \theta) = -\frac{\sqrt{3}}{2}$$

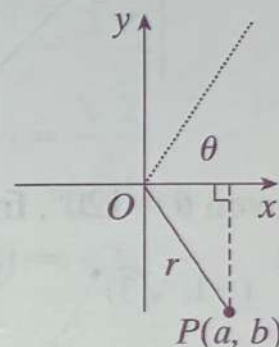
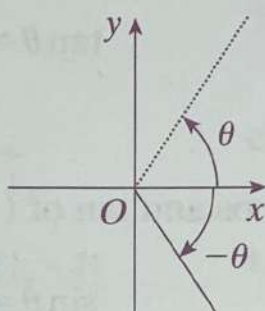
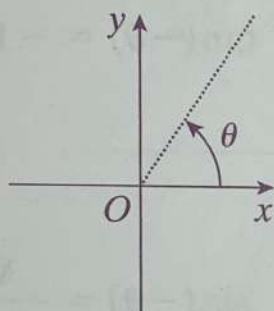
$$\tan \theta = \frac{\sqrt{3}}{3} \quad \tan(360^\circ - \theta) = -\frac{\sqrt{3}}{3}$$

Trigonometric Functions 3

Time : to : Date Name

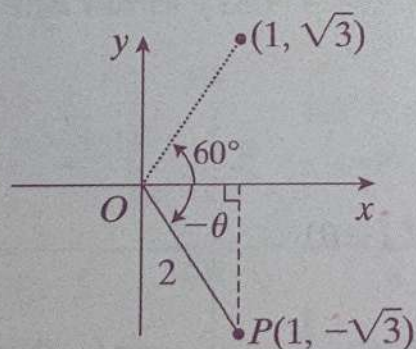
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For every angle θ , there is an angle of equal measure in the opposite direction, $-\theta$.



Ex.

Given $\theta = 60^\circ$, find the sin, cos and tan of $(-\theta)$.



$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\sin(-\theta) = -\frac{\sqrt{3}}{2}$$

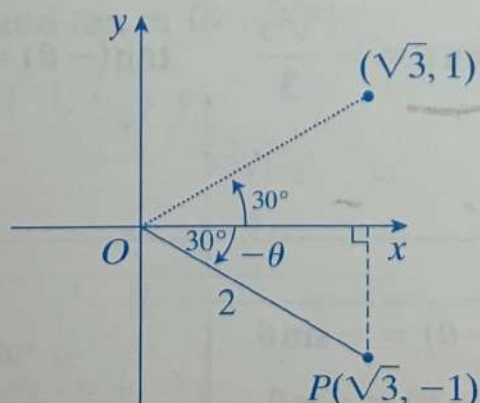
$$\cos \theta = \frac{1}{2}$$

$$\cos(-\theta) = \frac{1}{2}$$

$$\tan \theta = \sqrt{3}$$

$$\tan(-\theta) = -\sqrt{3}$$

1. Given $\theta = 30^\circ$, find the sin, cos and tan of $(-\theta)$.



$$\sin \theta = \frac{1}{2}$$

$$\sin(-\theta) = -\frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

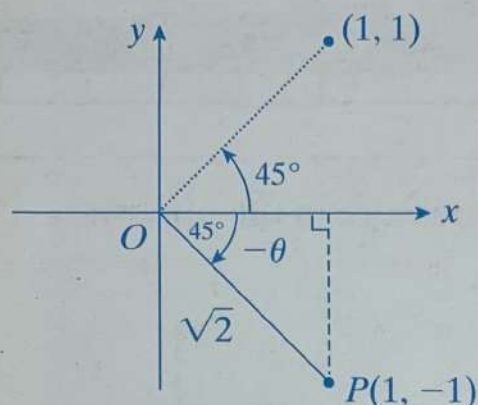
$$\cos(-\theta) = \frac{\sqrt{3}}{2}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$

$$\tan(-\theta) = -\frac{\sqrt{3}}{3}$$

M 26 b

2. Given $\theta = 45^\circ$, find the sin, cos and tan of $(-\theta)$.



$$\sin \theta = \frac{\sqrt{2}}{2}$$

$$\sin(-\theta) = -\frac{\sqrt{2}}{2}$$

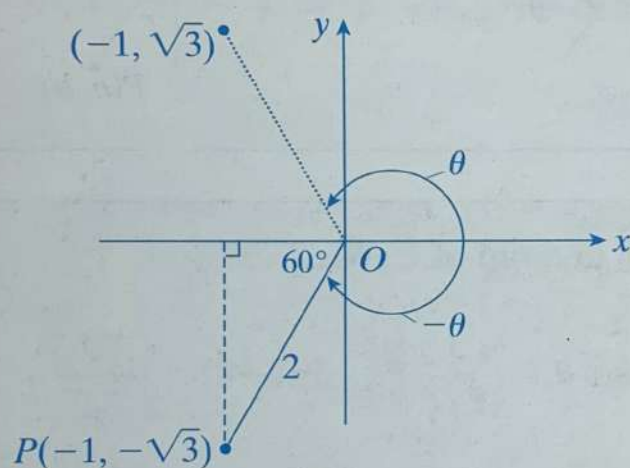
$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\cos(-\theta) = \frac{\sqrt{2}}{2}$$

$$\tan \theta = 1$$

$$\tan(-\theta) = -1$$

3. Given $\theta = 120^\circ$, find the sin, cos and tan of $(-\theta)$.



$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\sin(-\theta) = -\frac{\sqrt{3}}{2}$$

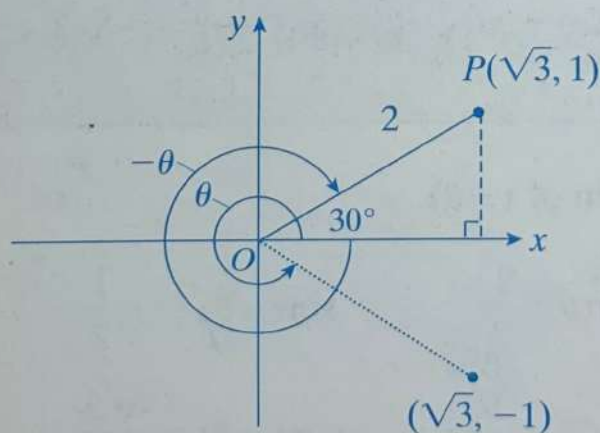
$$\cos \theta = -\frac{1}{2}$$

$$\cos(-\theta) = -\frac{1}{2}$$

$$\tan \theta = -\sqrt{3}$$

$$\tan(-\theta) = \sqrt{3}$$

4. Given $\theta = 330^\circ$, find the sin, cos and tan of $(-\theta)$.



$$\sin \theta = -\frac{1}{2}$$

$$\sin(-\theta) = \frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\cos(-\theta) = \frac{\sqrt{3}}{2}$$

$$\tan \theta = -\frac{\sqrt{3}}{3}$$

$$\tan(-\theta) = \frac{\sqrt{3}}{3}$$

Formulas

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

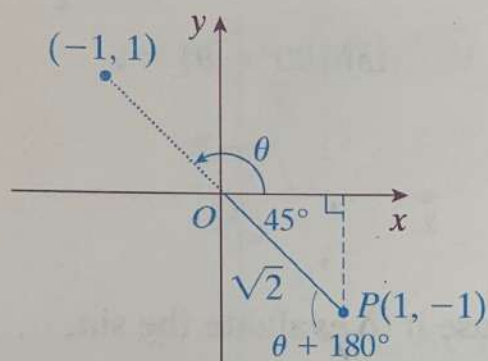
M 27 a

Trigonometric Functions 3

Time : to : Date Name

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1. Given $\theta = 135^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(\theta + 180^\circ)$.

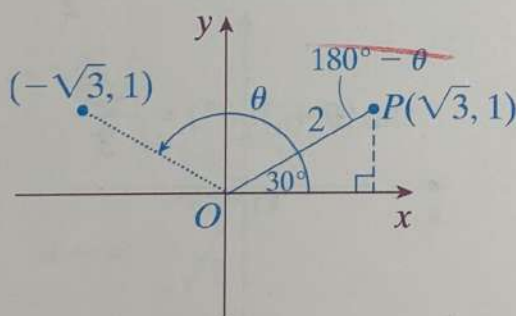


$$\sin(\theta + 180^\circ) = -\frac{\sqrt{2}}{2}$$

$$\cos(\theta + 180^\circ) = \frac{\sqrt{2}}{2}$$

$$\tan(\theta + 180^\circ) = -1$$

2. Given $\theta = 150^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(180^\circ - \theta)$.

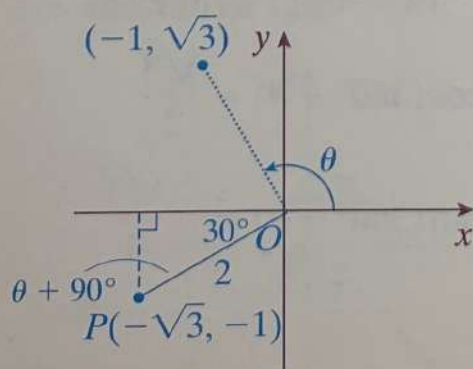


$$\sin(180^\circ - \theta) = \frac{1}{2}$$

$$\cos(180^\circ - \theta) = \frac{\sqrt{3}}{2}$$

$$\tan(180^\circ - \theta) = \frac{\sqrt{3}}{3}$$

3. Given $\theta = 120^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(\theta + 90^\circ)$.



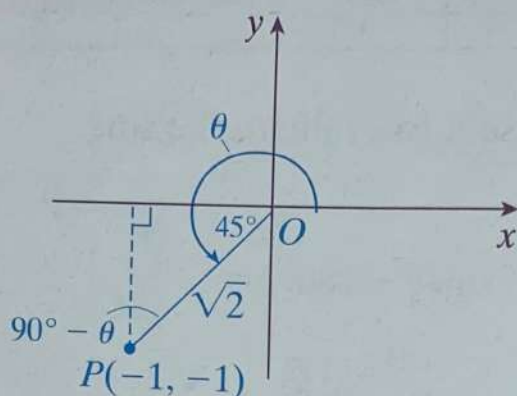
$$\sin(\theta + 90^\circ) = -\frac{1}{2}$$

$$\cos(\theta + 90^\circ) = -\frac{\sqrt{3}}{2}$$

$$\tan(\theta + 90^\circ) = \frac{\sqrt{3}}{3}$$

M 27 b

4. Given $\theta = 225^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(90^\circ - \theta)$.

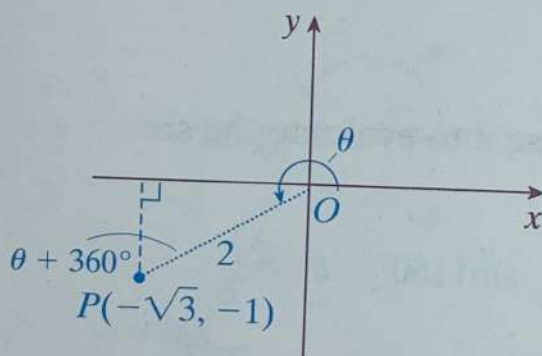


$$\sin(90^\circ - \theta) = -\frac{\sqrt{2}}{2}$$

$$\cos(90^\circ - \theta) = -\frac{\sqrt{2}}{2}$$

$$\tan(90^\circ - \theta) = 1$$

5. Given $\theta = 210^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(\theta + 360^\circ)$.

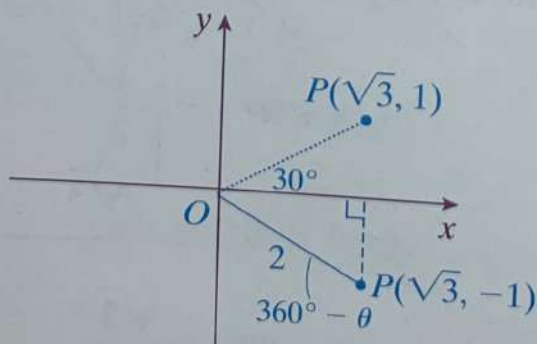


$$\sin(\theta + 360^\circ) = -\frac{1}{2}$$

$$\cos(\theta + 360^\circ) = -\frac{\sqrt{3}}{2}$$

$$\tan(\theta + 360^\circ) = \frac{\sqrt{3}}{3}$$

6. Given $\theta = 30^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(360^\circ - \theta)$.



$$\sin(360^\circ - \theta) = -\frac{1}{2}$$

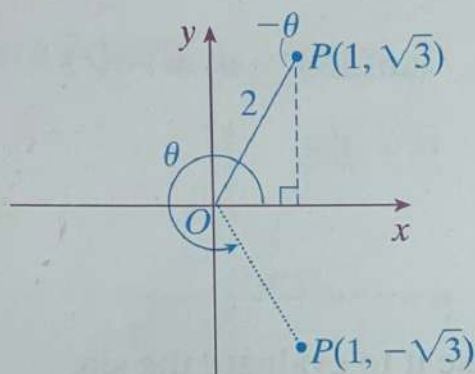
$$\cos(360^\circ - \theta) = \frac{\sqrt{3}}{2}$$

$$\tan(360^\circ - \theta) = -\frac{\sqrt{3}}{3}$$

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	1	-	2~

1. Given $\theta = 300^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(-\theta)$.

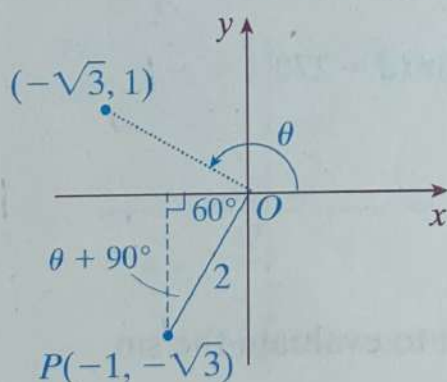


$$\sin(-\theta) = \frac{\sqrt{3}}{2}$$

$$\cos(-\theta) = \frac{1}{2}$$

$$\tan(-\theta) = \sqrt{3}$$

2. Given $\theta = 150^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(\theta + 90^\circ)$.

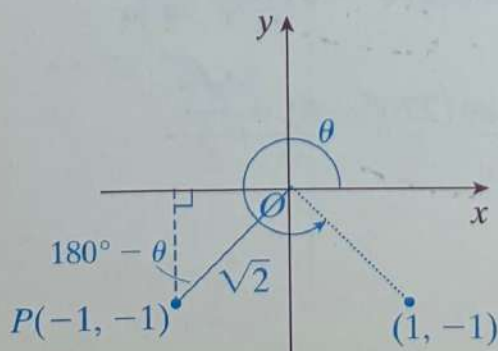


$$\sin(\theta + 90^\circ) = -\frac{\sqrt{3}}{2}$$

$$\cos(\theta + 90^\circ) = -\frac{1}{2}$$

$$\tan(\theta + 90^\circ) = \sqrt{3}$$

3. Given $\theta = 315^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(180^\circ - \theta)$.



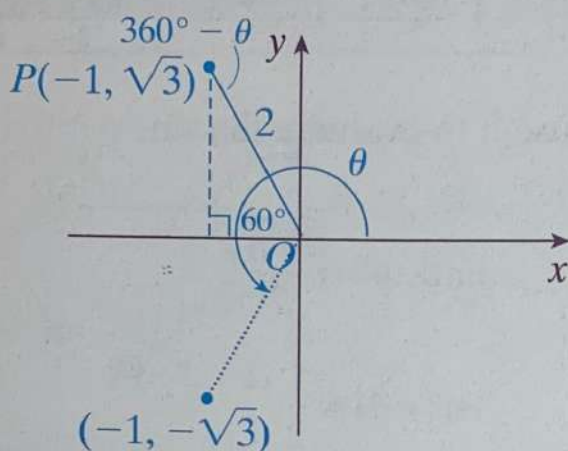
$$\sin(180^\circ - \theta) = -\frac{\sqrt{2}}{2}$$

$$\cos(180^\circ - \theta) = -\frac{\sqrt{2}}{2}$$

$$\tan(180^\circ - \theta) = 1$$

M 28 b

4. Given $\theta = 240^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(360^\circ - \theta)$.

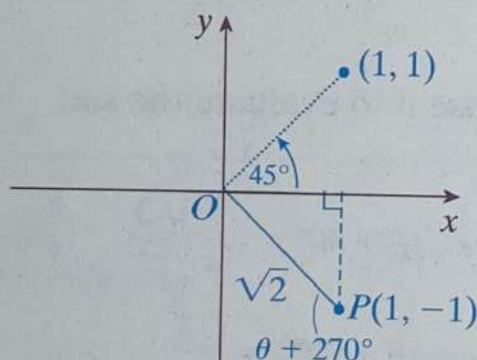


$$\sin(360^\circ - \theta) = \frac{\sqrt{3}}{2}$$

$$\cos(360^\circ - \theta) = -\frac{1}{2}$$

$$\tan(360^\circ - \theta) = -\sqrt{3}$$

5. Given $\theta = 45^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(\theta + 270^\circ)$.

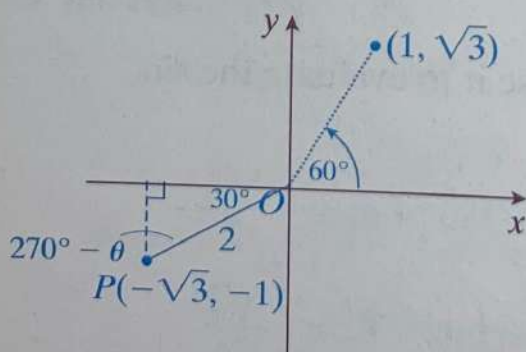


$$\sin(\theta + 270^\circ) = -\frac{\sqrt{2}}{2}$$

$$\cos(\theta + 270^\circ) = \frac{\sqrt{2}}{2}$$

$$\tan(\theta + 270^\circ) = -1$$

6. Given $\theta = 60^\circ$, draw the diagram and use it to evaluate the sin, cos and tan of $(270^\circ - \theta)$.



$$\sin(270^\circ - \theta) = -\frac{1}{2}$$

$$\cos(270^\circ - \theta) = -\frac{\sqrt{3}}{2}$$

$$\tan(270^\circ - \theta) = \frac{\sqrt{3}}{3}$$

M 29 a

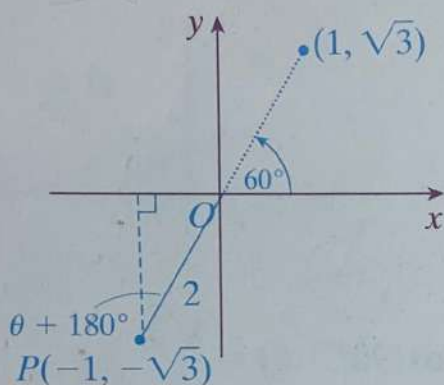
Trigonometric Functions 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	1	-	2~

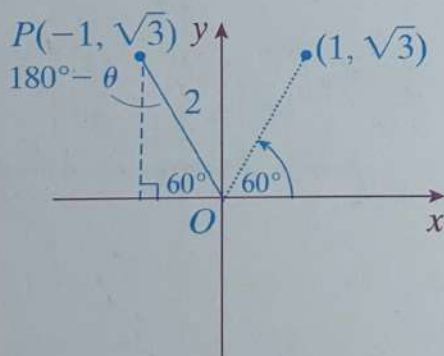
In each exercise, given that $\theta = 60^\circ$, draw the diagram and use it to evaluate the given trigonometric expression.

(1) $\sin(\theta + 180^\circ)$



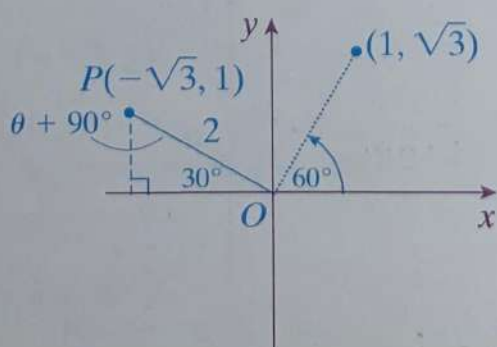
$$\sin(\theta + 180^\circ) = -\frac{\sqrt{3}}{2}$$

(2) $\sin(180^\circ - \theta)$



$$\sin(180^\circ - \theta) = \frac{\sqrt{3}}{2}$$

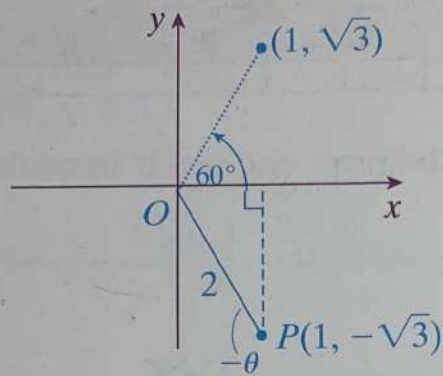
(3) $\sin(\theta + 90^\circ)$



$$\sin(\theta + 90^\circ) = \frac{1}{2}$$

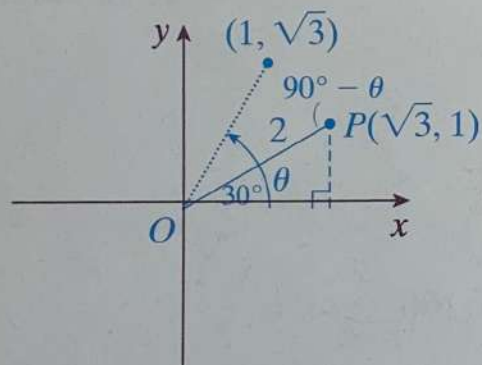
M 29 b

(4) $\cos(-\theta)$



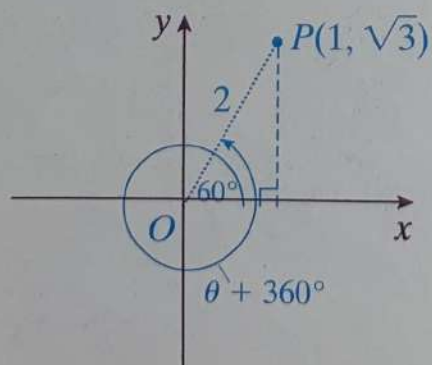
$$\cos(-\theta) = \frac{1}{2}$$

(5) $\cos(90^\circ - \theta)$



$$\cos(90^\circ - \theta) = \frac{\sqrt{3}}{2}$$

(6) $\cos(\theta + 360^\circ)$



$$\cos(\theta + 360^\circ) = \frac{1}{2}$$

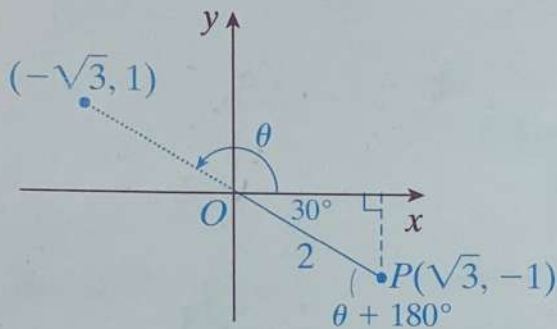
Trigonometric Functions 3

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	1	-	2~

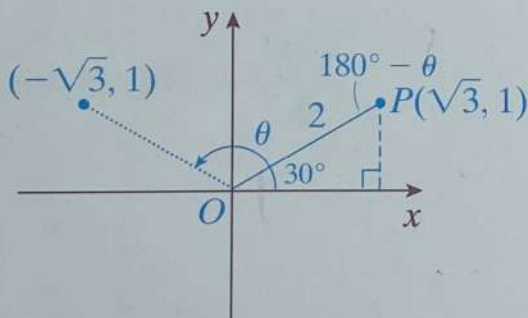
In each exercise, given that $\theta = 150^\circ$, draw the diagram and use it to evaluate the given trigonometric expression.

(1) $\cos(\theta + 180^\circ)$



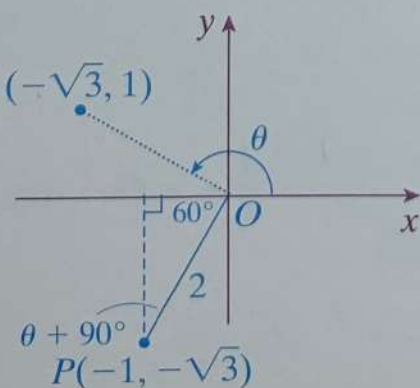
$$\cos(\theta + 180^\circ) = \frac{\sqrt{3}}{2}$$

(2) $\tan(180^\circ - \theta)$



$$\tan(180^\circ - \theta) = \frac{\sqrt{3}}{3}$$

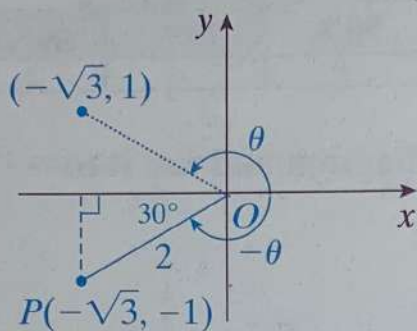
(3) $\cos(\theta + 90^\circ)$



$$\cos(\theta + 90^\circ) = -\frac{1}{2}$$

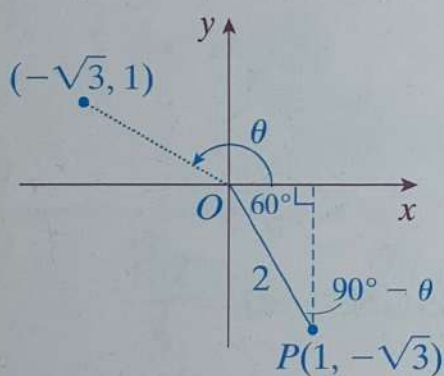
M 30 b

(4) $\tan(-\theta)$



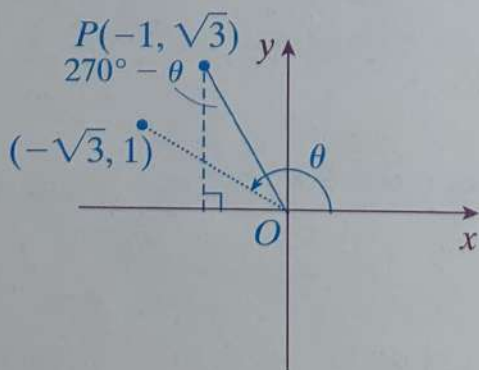
$$\tan(-\theta) = \frac{\sqrt{3}}{3}$$

(5) $\sin(90^\circ - \theta)$



$$\sin(90^\circ - \theta) = -\frac{\sqrt{3}}{2}$$

(6) $\sin(270^\circ - \theta)$



$$\sin(270^\circ - \theta) = \frac{\sqrt{3}}{2}$$

Trigonometric Functions 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

1. Given that $\theta = 30^\circ$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

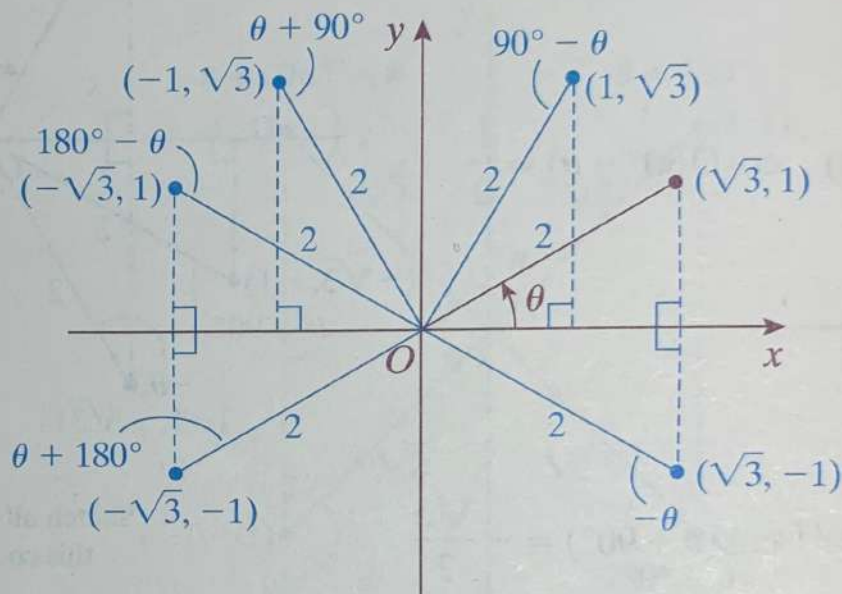
(1) $\sin(\theta + 180^\circ) = -\frac{1}{2}$

(2) $\sin(180^\circ - \theta) = \frac{1}{2}$

(3) $\sin(\theta + 90^\circ) = \frac{\sqrt{3}}{2}$

(4) $\sin(90^\circ - \theta) = \frac{\sqrt{3}}{2}$

(5) $\sin(-\theta) = -\frac{1}{2}$



Sketch all the diagrams on this coordinate grid.

M 31 b

2. Given that $\theta = 120^\circ$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

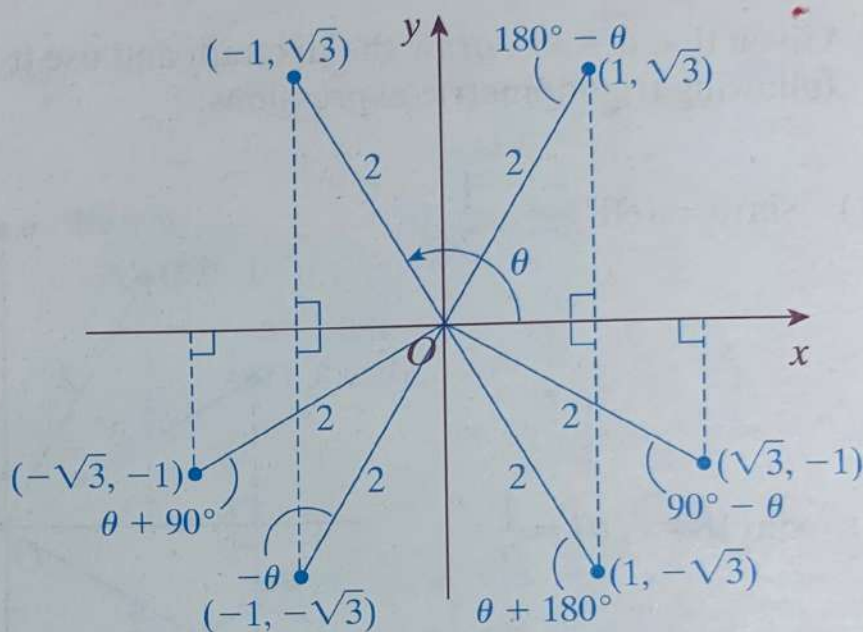
(1) $\cos(\theta + 180^\circ) = \frac{1}{2}$

(2) $\cos(180^\circ - \theta) = \frac{1}{2}$

(3) $\cos(\theta + 90^\circ) = -\frac{\sqrt{3}}{2}$

(4) $\cos(90^\circ - \theta) = \frac{\sqrt{3}}{2}$

(5) $\cos(-\theta) = -\frac{1}{2}$



Sketch all the diagrams on this coordinate grid.

Trigonometric Functions 4

Time : to : Date Name

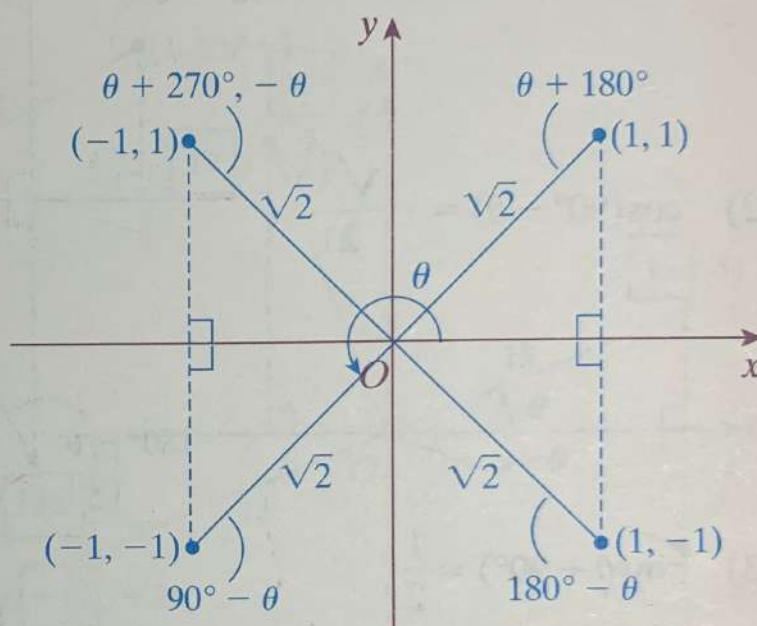
100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

1. Given that $\theta = 225^\circ$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

(1) $\cos(\theta + 180^\circ) = \frac{\sqrt{2}}{2}$

(2) $\tan(180^\circ - \theta) = -1$

(3) $\sin(\theta + 270^\circ) = \frac{\sqrt{2}}{2}$



Sketch all the diagrams on this coordinate grid.

(4) $\tan(90^\circ - \theta) = 1$

(5) $\cos(-\theta) = -\frac{\sqrt{2}}{2}$

M 32 b

2. Given that $\theta = 300^\circ$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

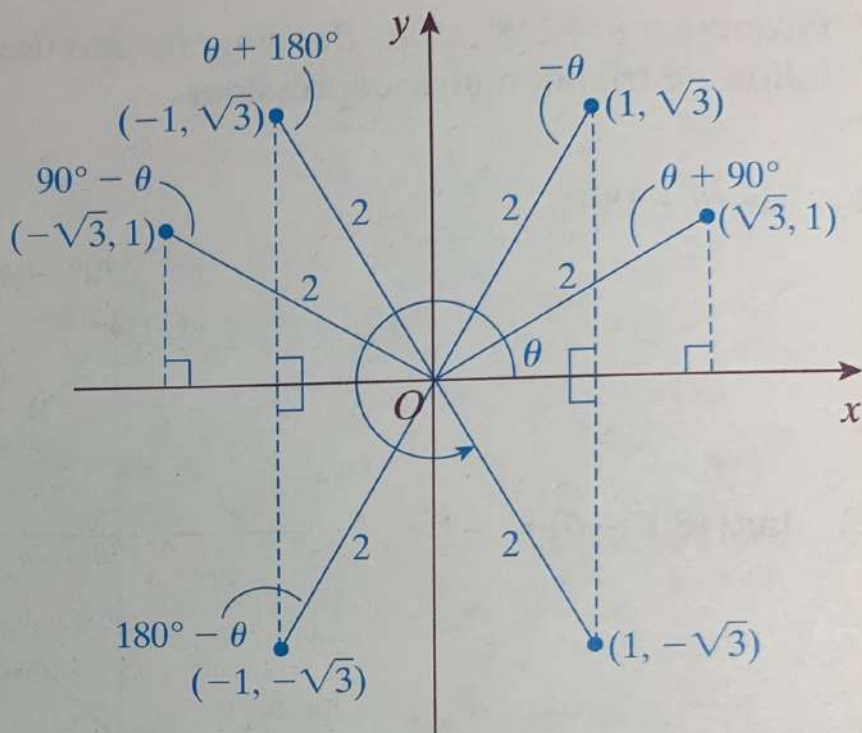
(1) $\sin(\theta + 180^\circ) = \frac{\sqrt{3}}{2}$

(2) $\cos(90^\circ - \theta) = -\frac{\sqrt{3}}{2}$

(3) $\sin(\theta + 90^\circ) = \frac{1}{2}$

(4) $\cos(180^\circ - \theta) = -\frac{1}{2}$

(5) $\tan(-\theta) = \sqrt{3}$



Sketch all the diagrams on this coordinate grid.

Trigonometric Functions 4

Time : to : Date Name

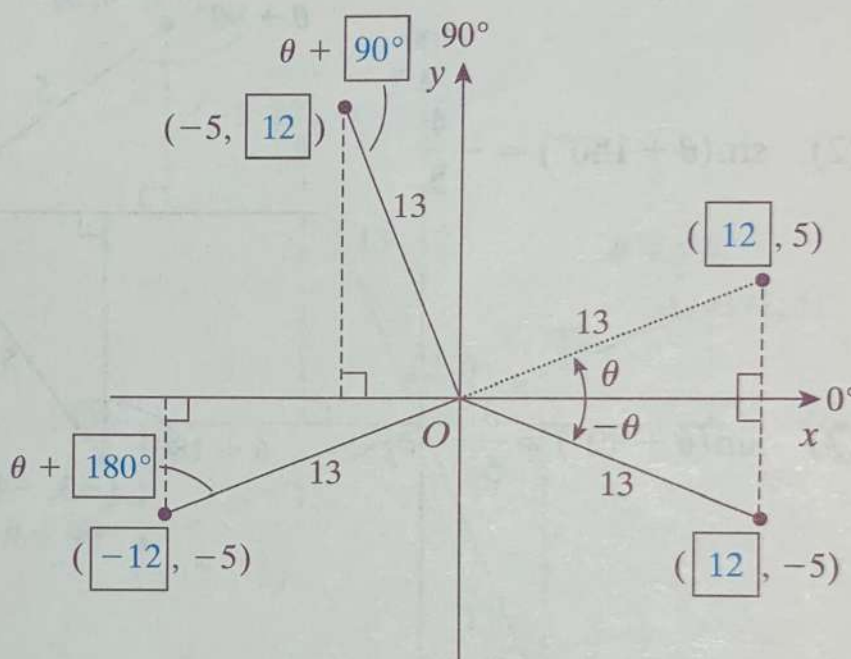
100%	90%	80%	70%	69%~
(mistakes) 0	1	2	3	4~

1. Given that $0^\circ < \theta < 90^\circ$, and $\sin \theta = \frac{5}{13}$, complete the diagram and use it to evaluate each of the following trigonometric expressions.

(1) $\sin(-\theta) = -\frac{5}{13}$

(2) $\sin(\theta + 180^\circ) = -\frac{5}{13}$

(3) $\sin(\theta + 90^\circ) = \frac{12}{13}$



(4) $\cos(-\theta) = \frac{12}{13}$

(5) $\cos(\theta + 180^\circ) = -\frac{12}{13}$

(6) $\cos(\theta + 90^\circ) = -\frac{5}{13}$

M 33 b

2. Given that $0^\circ < \theta < 90^\circ$, and $\cos \theta = \frac{3}{5}$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

(1) $\sin(-\theta) = -\frac{4}{5}$

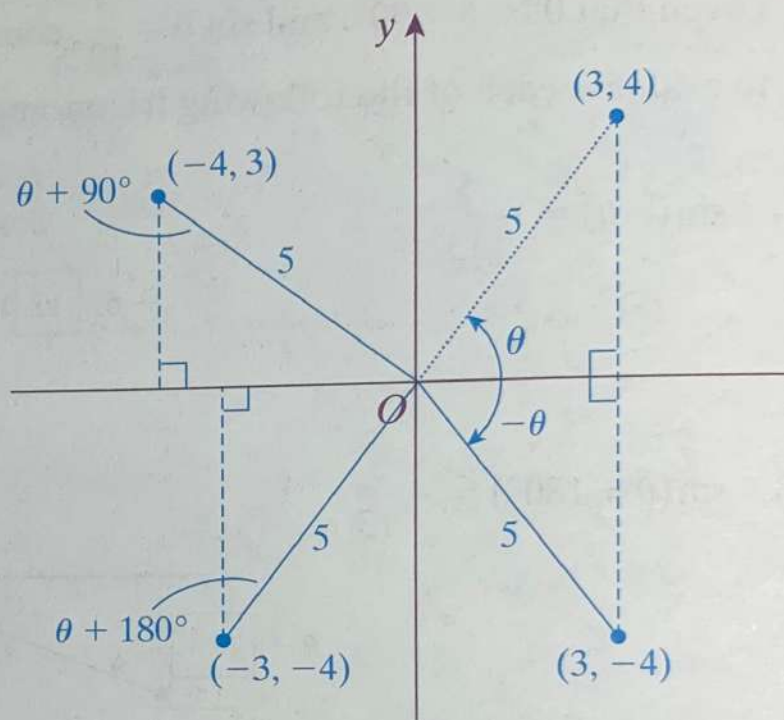
(2) $\sin(\theta + 180^\circ) = -\frac{4}{5}$

(3) $\sin(\theta + 90^\circ) = \frac{3}{5}$

(4) $\cos(-\theta) = \frac{3}{5}$

(5) $\cos(\theta + 180^\circ) = -\frac{3}{5}$

(6) $\cos(\theta + 90^\circ) = -\frac{4}{5}$



M 34 a

Trigonometric Functions 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2~3	4~5	6~

1. Given that $90^\circ < \theta < 180^\circ$, and $\cos \theta = -\frac{5}{13}$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

(1) $\sin(\theta + 90^\circ) = -\frac{5}{13}$

(2) $\sin(\theta + 180^\circ) = -\frac{12}{13}$

(3) $\sin(\theta + 270^\circ) = \frac{5}{13}$

(4) $\cos(\theta + 90^\circ) = -\frac{12}{13}$

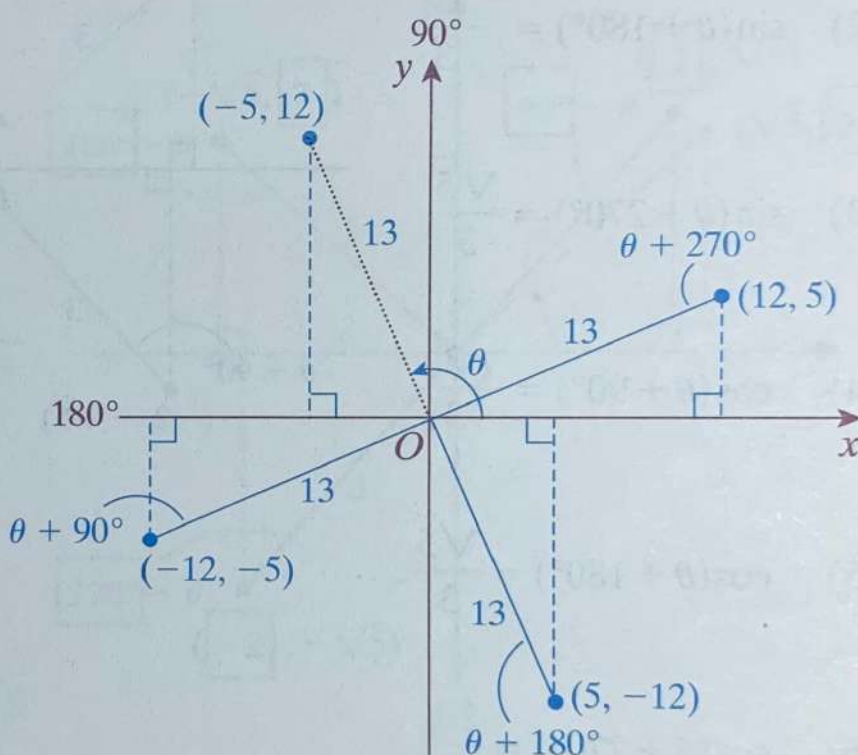
(5) $\cos(\theta + 180^\circ) = \frac{5}{13}$

(6) $\cos(\theta + 270^\circ) = \frac{12}{13}$

(7) $\tan(\theta + 90^\circ) = \frac{5}{12}$

(8) $\tan(\theta + 180^\circ) = -\frac{12}{5}$

(9) $\tan(\theta + 270^\circ) = \frac{5}{12}$



M 34 b

2. Given that $90^\circ < \theta < 180^\circ$, and $\sin \theta = \frac{2}{3}$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

$$(1) \quad \sin(\theta + 90^\circ) = -\frac{\sqrt{5}}{3}$$

$$(2) \quad \sin(\theta + 180^\circ) = -\frac{2}{3}$$

$$(3) \quad \sin(\theta + 270^\circ) = \frac{\sqrt{5}}{3}$$

$$(4) \quad \cos(\theta + 90^\circ) = -\frac{2}{3}$$

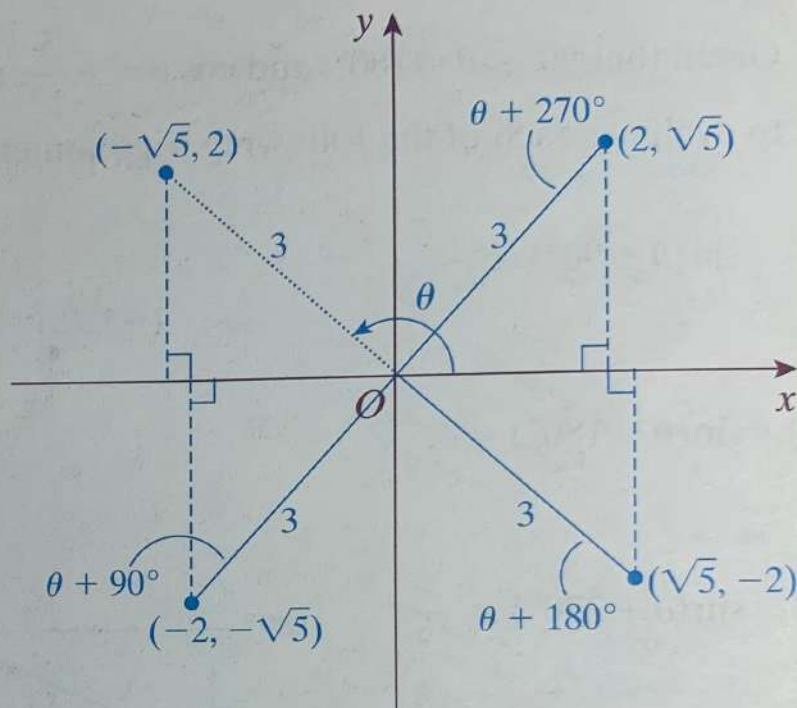
$$(5) \quad \cos(\theta + 180^\circ) = -\frac{\sqrt{5}}{3}$$

$$(6) \quad \cos(\theta + 270^\circ) = \frac{2}{3}$$

$$(7) \quad \tan(\theta + 90^\circ) = \frac{\sqrt{5}}{2}$$

$$(8) \quad \tan(\theta + 180^\circ) = -\frac{2\sqrt{5}}{5}$$

$$(9) \quad \tan(\theta + 270^\circ) = \frac{\sqrt{5}}{2}$$



Trigonometric Functions 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2~3	4~5	6~

1. Given that $0^\circ < \theta < 90^\circ$, and $\cos \theta = \frac{\sqrt{5}}{3}$, complete the diagram and use it to evaluate each of the following trigonometric expressions.

(1) $\sin(90^\circ - \theta) = \frac{\sqrt{5}}{3}$

(2) $\sin(180^\circ - \theta) = \frac{2}{3}$

(3) $\sin(270^\circ - \theta) = -\frac{\sqrt{5}}{3}$

(4) $\cos(90^\circ - \theta) = \frac{2}{3}$

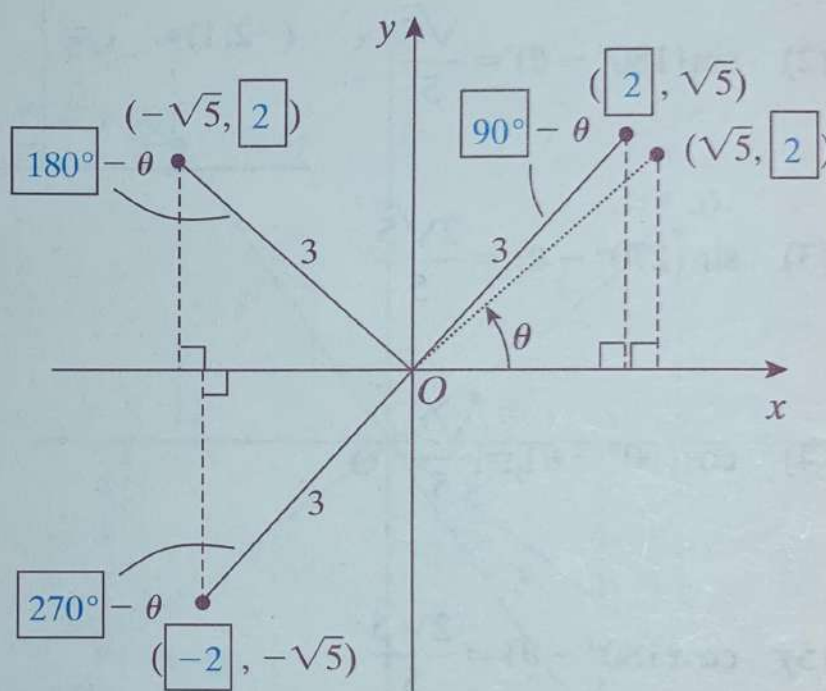
(5) $\cos(180^\circ - \theta) = -\frac{\sqrt{5}}{3}$

(6) $\cos(270^\circ - \theta) = -\frac{2}{3}$

(7) $\tan(90^\circ - \theta) = \frac{\sqrt{5}}{2}$

(8) $\tan(180^\circ - \theta) = -\frac{2\sqrt{5}}{5}$

(9) $\tan(270^\circ - \theta) = \frac{\sqrt{5}}{2}$



M 35 b

2. Given that $90^\circ < \theta < 180^\circ$, and $\tan \theta = -\frac{1}{2}$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

(1) $\sin(90^\circ - \theta) = -\frac{2\sqrt{5}}{5}$

(2) $\sin(180^\circ - \theta) = \frac{\sqrt{5}}{5}$

(3) $\sin(270^\circ - \theta) = \frac{2\sqrt{5}}{5}$

(4) $\cos(90^\circ - \theta) = \frac{\sqrt{5}}{5}$

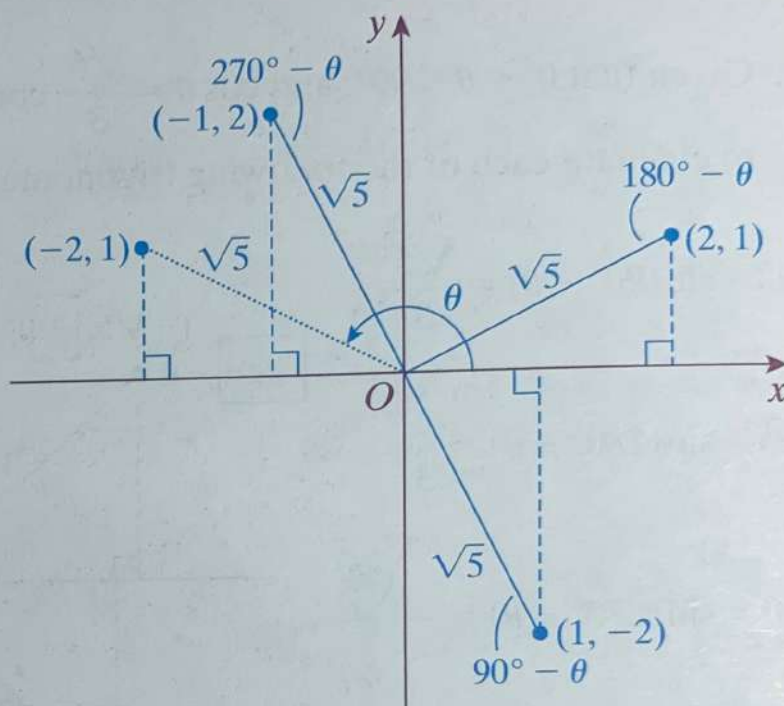
(5) $\cos(180^\circ - \theta) = \frac{2\sqrt{5}}{5}$

(6) $\cos(270^\circ - \theta) = -\frac{\sqrt{5}}{5}$

(7) $\tan(90^\circ - \theta) = -2$

(8) $\tan(180^\circ - \theta) = \frac{1}{2}$

(9) $\tan(270^\circ - \theta) = -2$



M 36 a

Trigonometric Functions 4

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1~2	3~4	5~6	7~

Given that $0^\circ < \theta < 90^\circ$, and $\sin \theta = \frac{3}{5}$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

(1) $\sin(-\theta) = -\frac{3}{5}$

(2) $\sin(\theta + 90^\circ) = \frac{4}{5}$

(3) $\sin(\theta + 180^\circ) = -\frac{3}{5}$

(4) $\sin(\theta + 270^\circ) = -\frac{4}{5}$

(5) $\cos \theta = \frac{4}{5}$

(6) $\cos(-\theta) = \frac{4}{5}$

(7) $\cos(\theta + 90^\circ) = -\frac{3}{5}$

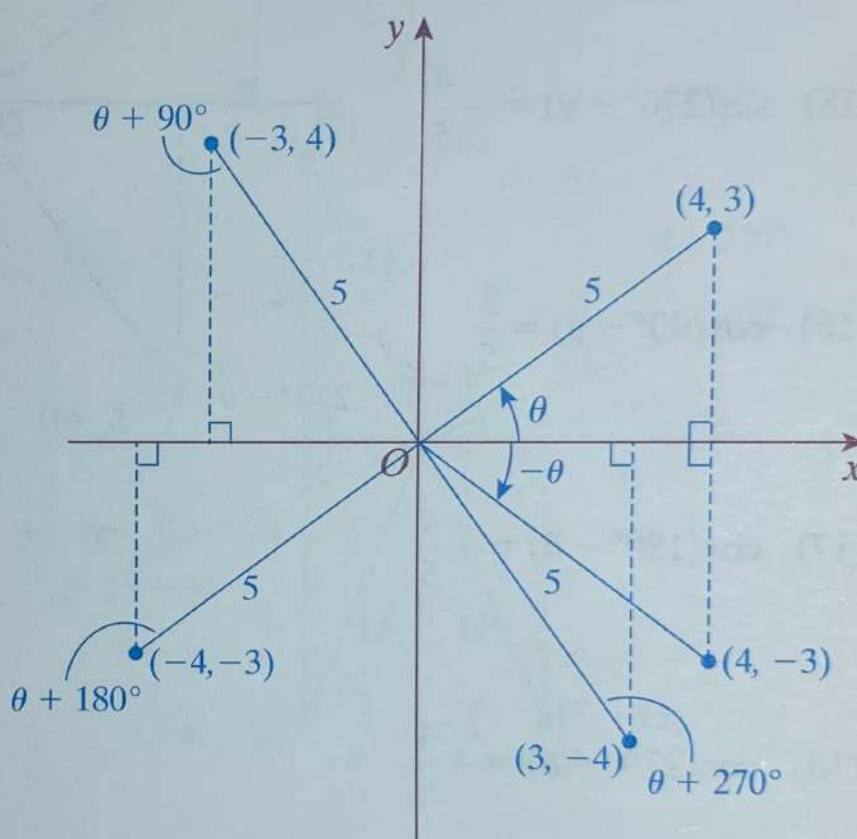
(8) $\cos(\theta + 180^\circ) = -\frac{4}{5}$

(9) $\cos(\theta + 270^\circ) = \frac{3}{5}$

(10) $\tan \theta = \frac{3}{4}$

(11) $\tan(\theta + 90^\circ) = -\frac{4}{3}$

(12) $\tan(\theta + 180^\circ) = \frac{3}{4}$



M 36 b

$$(13) \quad \sin(90^\circ - \theta) = \frac{4}{5}$$

$$(14) \quad \sin(180^\circ - \theta) = \frac{3}{5}$$

$$(15) \quad \sin(270^\circ - \theta) = -\frac{4}{5}$$

$$(16) \quad \cos(90^\circ - \theta) = \frac{3}{5}$$

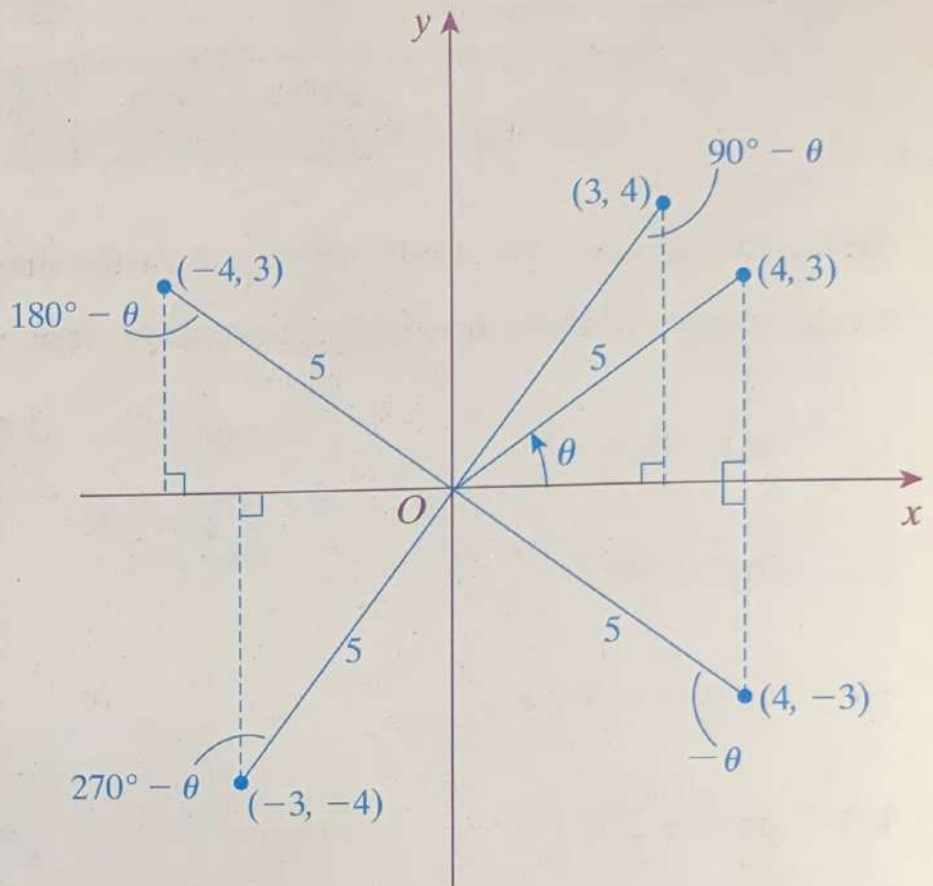
$$(17) \quad \cos(180^\circ - \theta) = -\frac{4}{5}$$

$$(18) \quad \cos(270^\circ - \theta) = -\frac{3}{5}$$

$$(19) \quad \tan(-\theta) = -\frac{3}{4}$$

$$(20) \quad \tan(90^\circ - \theta) = \frac{4}{3}$$

$$(21) \quad \tan(180^\circ - \theta) = -\frac{3}{4}$$



Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1~2	3~4	5~6	7~

Given that $90^\circ < \theta < 180^\circ$, and $\tan \theta = -\frac{12}{5}$, draw the diagram and use it to evaluate each of the following trigonometric expressions.

(1) $\sin \theta = \frac{12}{13}$

(2) $\sin(-\theta) = -\frac{12}{13}$

(3) $\sin(\theta + 90^\circ) = -\frac{5}{13}$

(4) $\sin(\theta + 180^\circ) = -\frac{12}{13}$

(5) $\sin(\theta + 270^\circ) = \frac{5}{13}$

(6) $\cos \theta = -\frac{5}{13}$

(7) $\cos(-\theta) = -\frac{5}{13}$

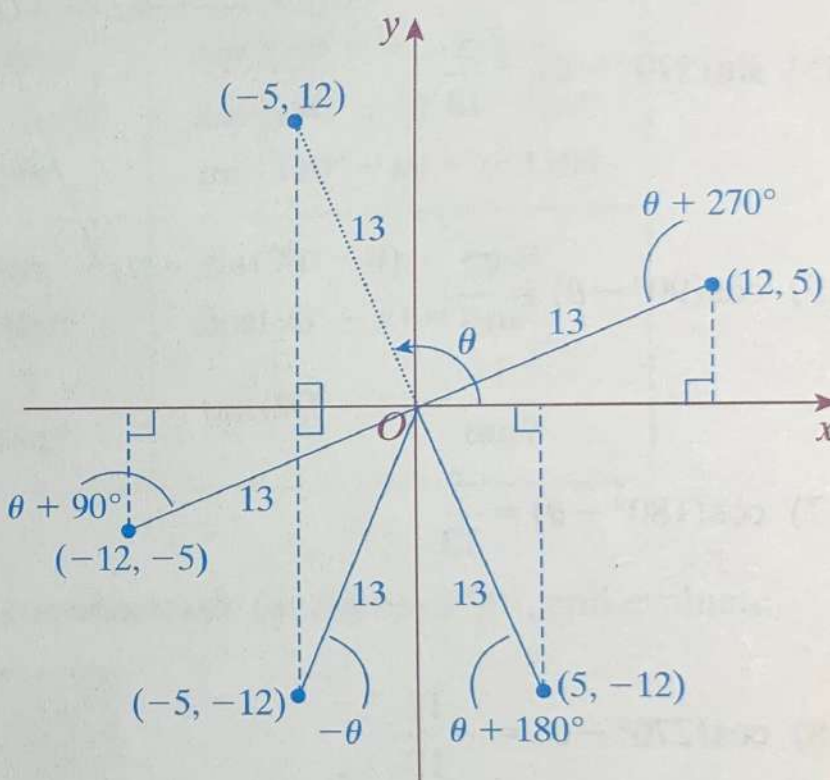
(8) $\cos(\theta + 90^\circ) = -\frac{12}{13}$

(9) $\cos(\theta + 180^\circ) = \frac{5}{13}$

(10) $\cos(\theta + 270^\circ) = \frac{12}{13}$

(11) $\tan(\theta + 90^\circ) = \frac{5}{12}$

(12) $\tan(\theta + 180^\circ) = -\frac{12}{5}$



M 37 b

$$(13) \sin(90^\circ - \theta) = \frac{5}{13}$$

$$(14) \sin(180^\circ - \theta) = \frac{12}{13}$$

$$(15) \sin(270^\circ - \theta) = \frac{5}{13}$$

$$(16) \cos(90^\circ - \theta) = \frac{12}{13}$$

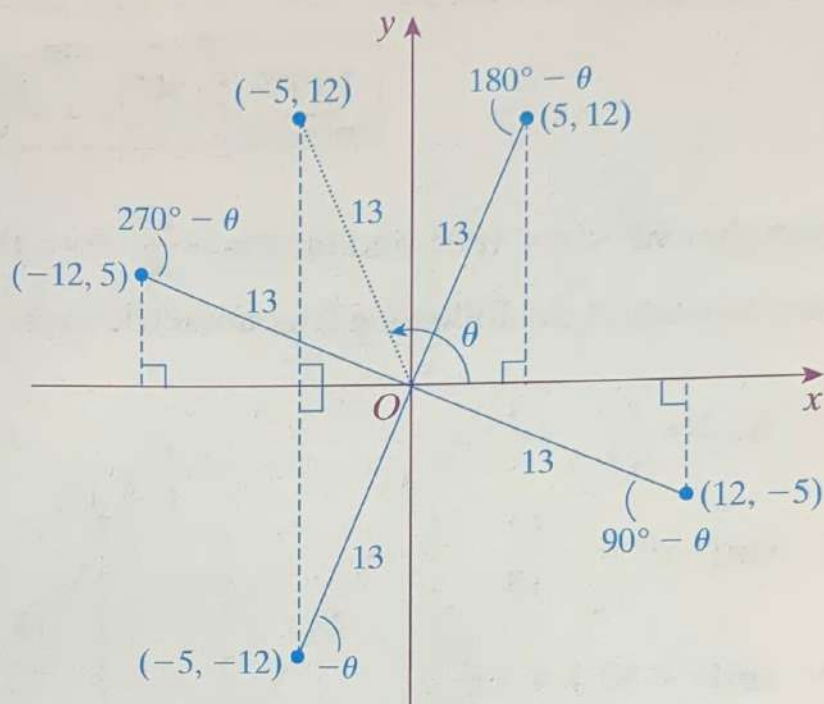
$$(17) \cos(180^\circ - \theta) = -\frac{5}{13}$$

$$(18) \cos(270^\circ - \theta) = -\frac{12}{13}$$

$$(19) \tan(-\theta) = \frac{12}{5}$$

$$(20) \tan(90^\circ - \theta) = -\frac{5}{12}$$

$$(21) \tan(180^\circ - \theta) = \frac{12}{5}$$



Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	1	2~3	4~5	6~

Review Formulas	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$
$\sin(\theta + 180^\circ) = -\sin\theta$ $\cos(\theta + 180^\circ) = -\cos\theta$ $\tan(\theta + 180^\circ) = \tan\theta$	$\sin(180^\circ - \theta) = \sin\theta$ $\cos(180^\circ - \theta) = -\cos\theta$ $\tan(180^\circ - \theta) = -\tan\theta$
$\sin(\theta + 90^\circ) = \cos\theta$ $\cos(\theta + 90^\circ) = -\sin\theta$ $\tan(\theta + 90^\circ) = -\frac{1}{\tan\theta}$	$\sin(90^\circ - \theta) = \cos\theta$ $\cos(90^\circ - \theta) = \sin\theta$ $\tan(90^\circ - \theta) = \frac{1}{\tan\theta}$

1. Express the following as trigonometric functions of 30° , and evaluate.

Ex.

$$\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$(1) \quad \cos 60^\circ = \sin 30^\circ = \frac{1}{2}$$

$$(4) \quad \tan 150^\circ = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$$

$$(2) \quad \tan 120^\circ = -\tan 60^\circ$$

$$(5) \quad \cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$= -\frac{1}{\tan 30^\circ} = -\sqrt{3}$$

$$(3) \quad \sin 120^\circ = \sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

M 38 b

2. Rewrite each of the following as a trigonometric expression of a positive angle less than 45° .

Ex.

$$\sin 72^\circ = \sin(90^\circ - 18^\circ) = \cos 18^\circ$$

$$(1) \quad \cos 78^\circ = \cos(90^\circ - 12^\circ) = \sin 12^\circ$$

$$(2) \quad \tan 65^\circ = \tan(90^\circ - 25^\circ) = \frac{1}{\tan 25^\circ}$$

$$(3) \quad \sin 134^\circ = \sin(90^\circ + 44^\circ) = \cos 44^\circ$$

$$(4) \quad \cos 165^\circ = \cos(180^\circ - 15^\circ) = -\cos 15^\circ$$

$$(5) \quad \tan 100^\circ = \tan(90^\circ + 10^\circ) = -\frac{1}{\tan 10^\circ}$$

$$(6) \quad \sin 168^\circ = \sin(180^\circ - 12^\circ) = \sin 12^\circ$$

$$(7) \quad \cos 106^\circ = \cos(90^\circ + 16^\circ) = -\sin 16^\circ$$

$$(8) \quad \tan 224^\circ = \tan(180^\circ + 44^\circ) = \tan 44^\circ$$

3. Evaluate the following expressions.

$$(1) \quad \sin(-390^\circ) = -\sin 390^\circ = -\sin 30^\circ = \boxed{-\frac{1}{2}}$$

$$(2) \quad \cos(-120^\circ) = \cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$$

$$(3) \quad \cos(-300^\circ) = \cos 300^\circ = \cos 60^\circ = \frac{1}{2}$$

$$(4) \quad \tan(-225^\circ) = -\tan 225^\circ = -\tan 45^\circ = -1$$

$$(5) \quad \tan(-315^\circ) = -\tan 315^\circ = -(-\tan 45^\circ) = \tan 45^\circ = 1$$

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	1	-	2~

1. Simplify the following expressions.

Ex.

$$\begin{aligned}
 & \tan \alpha \sin(90^\circ - \alpha) - \cos^2 \alpha \cos(90^\circ - \alpha) \\
 &= \frac{\sin \alpha}{\cos \alpha} \cdot \cos \alpha - \cos^2 \alpha \sin \alpha \\
 &= \sin \alpha - \cos^2 \alpha \sin \alpha \\
 &= \sin \alpha (1 - \cos^2 \alpha) \quad \text{Factor.} \\
 &= \sin \alpha (\sin^2 \alpha) \quad \text{From } \sin^2 \alpha + \cos^2 \alpha = 1, \\
 & \quad \sin^2 \alpha = 1 - \cos^2 \alpha \\
 &= \sin^3 \alpha
 \end{aligned}$$

From: $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$
 $\sin(90^\circ - \alpha) = \cos \alpha$
 $\cos(90^\circ - \alpha) = \sin \alpha$

$$\begin{aligned}
 (1) \quad & (\sin \alpha + \cos \alpha)^2 + (\sin \alpha - \cos \alpha)^2 \\
 &= 1 + 2 \sin \alpha \cos \alpha + 1 - 2 \sin \alpha \cos \alpha \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad & \frac{1 - \sin \alpha}{1 + \sin \alpha} + \frac{1 + \sin \alpha}{1 - \sin \alpha} - 2 \\
 &= \frac{2 + 2 \sin^2 \alpha}{1 - \sin^2 \alpha} - 2 \\
 &= \frac{2 + 2 \sin^2 \alpha - 2 + 2 \sin^2 \alpha}{1 - \sin^2 \alpha} \\
 &= \frac{4 \sin^2 \alpha}{\cos^2 \alpha} \\
 &= 4 \tan^2 \alpha
 \end{aligned}$$

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2. Prove the following equalities.

$$(1) \quad 1 + \frac{1}{\tan^2 A} = \frac{1}{\sin^2 A}$$

$$\begin{aligned} [\text{Sol}] \text{ LHS} &= 1 + \frac{1}{\left(\frac{\sin A}{\cos A}\right)^2} = 1 + \frac{\cos^2 A}{\sin^2 A} \\ &= \frac{\sin^2 A + \cos^2 A}{\sin^2 A} = \frac{1}{\sin^2 A} = \text{RHS} \end{aligned}$$

$$(2) \quad \frac{\sin(90^\circ - A)}{\sin A} + \frac{\sin A}{\sin(90^\circ - A)} = \frac{1}{\sin A \cos A}$$

$$\begin{aligned} [\text{Sol}] \text{ LHS} &= \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A} = \frac{\cos^2 A + \sin^2 A}{\sin A \cos A} \\ &= \frac{1}{\sin A \cos A} = \text{RHS} \end{aligned}$$

$$(3) \quad \frac{1}{\tan^2(90^\circ - A)} + 1 = \frac{1}{\cos^2 A}$$

$$\begin{aligned} [\text{Sol}] \text{ LHS} &= \frac{\cos^2(90^\circ - A)}{\sin^2(90^\circ - A)} + 1 \\ &= \frac{\sin^2 A}{\cos^2 A} + 1 \\ &= \frac{\sin^2 A + \cos^2 A}{\cos^2 A} \\ &= \frac{1}{\cos^2 A} = \text{RHS} \end{aligned}$$

Time : to : Date Name

100%	90%	80%	70%	69%~
(mistakes) 0	-	1	-	2~

Prove the following equalities.

(1) $\tan^2 A - \tan^2 A \sin^2 A = \sin^2 A$

$$\begin{aligned}
 [\text{Sol}] \text{ LHS} &= \tan^2 A (1 - \sin^2 A) \\
 &= \frac{\sin^2 A}{\cos^2 A} \cdot \cos^2 A \\
 &= \sin^2 A = \text{RHS}
 \end{aligned}$$

(2) $\sin^4 A - \cos^4 A = 2\sin^2 A - 1$

$$\begin{aligned}
 [\text{Sol}] \text{ LHS} &= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A) \\
 &= \sin^2 A - \cos^2 A \\
 &= \sin^2 A - (1 - \sin^2 A) \\
 &= 2\sin^2 A - 1 = \text{RHS}
 \end{aligned}$$

(3) $\frac{1 + \sin A}{\cos A} + \frac{\cos A}{1 + \sin A} = \frac{2}{\cos A}$

$$\begin{aligned}
 [\text{Sol}] \text{ LHS} &= \frac{(1 + \sin A)^2 + \cos^2 A}{\cos A(1 + \sin A)} \\
 &= \frac{1 + 2\sin A + \sin^2 A + \cos^2 A}{\cos A(1 + \sin A)} \\
 &= \frac{2(1 + \sin A)}{\cos A(1 + \sin A)} = \frac{2}{\cos A} = \text{RHS}
 \end{aligned}$$

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$$(4) \quad \frac{\sin A}{1 + \cos A} + \frac{\sin A}{1 - \cos A} = \frac{2}{\sin A}$$

$$[\text{Sol}] \text{ LHS} = \frac{\sin A(1 - \cos A) + \sin A(1 + \cos A)}{(1 + \cos A)(1 - \cos A)}$$

$$= \frac{2 \sin A}{1 - \cos^2 A}$$

$$= \frac{2 \sin A}{\sin^2 A}$$

$$= \frac{2}{\sin A} = \text{RHS}$$

$$(5) \quad \frac{1}{1 + \sin A} + \frac{1}{1 - \sin A} = \frac{2}{\cos^2 A}$$

$$[\text{Sol}] \text{ LHS} = \frac{(1 - \sin A) + (1 + \sin A)}{(1 + \sin A)(1 - \sin A)}$$

$$= \frac{2}{1 - \sin^2 A}$$

$$= \frac{2}{\cos^2 A} = \text{RHS}$$

$$(6) \quad \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

$$[\text{Sol}] \text{ LHS} = \frac{1 - \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\sin^2 A}{\cos^2 A}}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$$

$$= \cos^2 A - \sin^2 A = \text{RHS}$$